A Bezout Computable Nonstandard Model of Open Induction

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Abstract

In contrast with Tennenbaum's theorem [8] that says Peano Arithmetic (PA) has no nonstandard computable model, Shepherdson [6] constructed a computable nonstandard model for a very weak fragment of PA, called Open Induction (Iopen) in which the induction scheme is only allowed to be applied for quantifier-free formulas (with parameters). Since then several attempts have been made from both sides to strengthen Tennenbaum's and Shepherdson's theorems. From one direction one would like to find fragments of arithmetic as weak as possible with no nonstandard computable model. On the other hand we are also interested in knowing those fragments that are as strong as possible and do have a computable nonstandard model. Attempts in the first direction were culminated in the work of Wilmers [10] where it is shown that IE_1 does not have a computable nonstandard model (IE_1 is the fragment based on the induction scheme for bounded existential formulas). Our work deals with the second direction. Since Open Induction is too weak to prove many true statements of number theory (It cannot even prove irrationality of $\sqrt{2}$), a number of algebraic first order properties have been suggested to be added to Iopen in order to obtain closer systems to number theory. These properties include: Normality [2], having the GCD property [7], being a Bezout domain [3], cofinality of primes (abbreviated here as cof(prime)) and so on. We mention that GCD is stronger than normality, Bezout is stronger than GCD and Bezout is weaker than IE_1. Berarducci and Otero [1], based on earlier works of Wilkie [9], van den Dries [2] and Macintyre-Marker [3] constructed a computable nonstandard model for Iopen + Normality + cof(prime). Also Moniri [5] by using transseries, managed to generalize Shepherdson's method directly, to construct primitive recursive nonstandard models of Iopen + cof(prime) with any finite transcendence degree > 1. In [4] we succeeded to strengthen Berarducci-Otero's construction by combining their method with that of Smith [7](which is itself a generalization of Macintyre-Marker's work to the GCD and Bezout case) and obtained a nonstandard computable model of Iopen+GCD+cof(prime). In this talk, we go one step further by bringing all of these materials together (Smith's chains, Berarducci-Otero's computable construction and Moniri's transseries) to produce a computable nonstandard model of Open Induction which is Bezout and has cofinal primes.

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