LOGIC COLLOQUIUM 2008

Bern, Switzerland

July 3rd - 8th

Gottlob Frege: *Begriffsschrift* and Impredicativity.

A Historical Note on Benno Kerry.

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Abstract

In the spirit of the 19^{th} Century formalization of analysis in terms of arithmetic, the German mathematician Gottlob Frege (1848-1926) aimed at an even more fundamental goal: to demonstrate that arithmetic itself could be properly expressed in terms of purely logical concepts. A completely new logical system was presented in order to reach this goal in (1879) *Begriffsschrift. Eine der arithmetischen nachgebildete Formelsprache des reinen Denken*. Despite its importance as a precise logical calculus, Frege's *Begriffsschrift* remained misunderstood and underappreciated. This annoyed Frege deeply. Even though, Frege himself seems to ignore that he had in the young Austrian philosopher Benno Kerry (1858-1889) the sharpest reader of the days. Between 1885 and 1890 appeared in a well known Journal of that time, *Vierteljahrsschrift für wissenschaftliche Philosophie*, a series of eight articles titled "Über Anschauung und ihre psychische Verarbeitung" by Kerry, articles certainly well known by Frege. In fact, on his (1892) "On Concept and Object" Frege recognizes that Kerry's criticisms to his notion of concept were the motivation for this famous article. But if we look into Kerry's work, we immediately realize that, being highly informed in the 19^{th} Century issues of the foundation of mathematics, his criticism goes far beyond Frege's technical notion of concept. In particular, he accused Frege's logicist definition of "following in a series" –which is in the core of the Frege's logicist reduction of the induction principle– of being circular.

In my work I aim to show that such criticism is an antecedent of the today known as "impredicativity problem", and that it is also an antecedent of Russell's *Vitiosus Circulus* Principle, which was formulated in 1908 as an eventual justification of his type theory. With such a purpose on mind, I offer in **I** an analysis of the Fregean definition of succession; in **II**, I present Kerry's criticisms and in **III** Russell's comments about it in his "Appendix A" of (1903) *Principles of Mathematics*. I leave **IV** for conclusions and some few bibliography remarks.

I. The Fregean Definition of "following in a series"

Let *f* be a binary relation. Previous definition of Hereditary Property (HP):

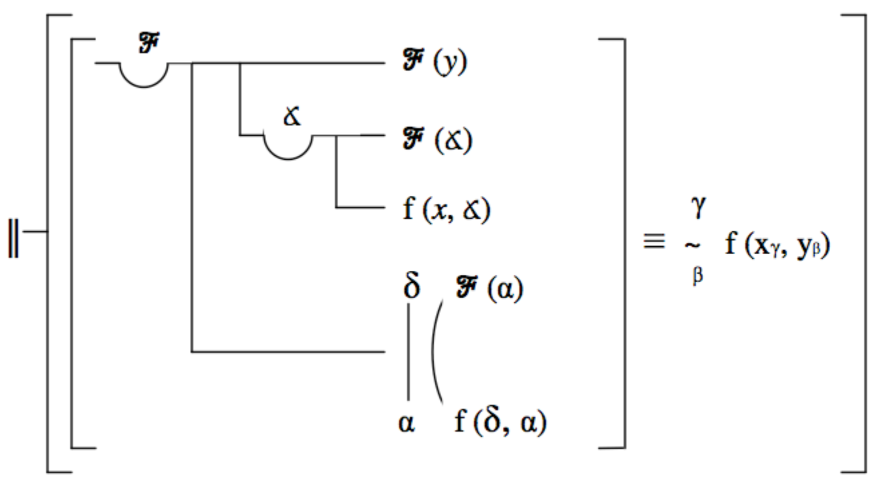
F is an **hereditary property w.r.t**. *f* iff $(\forall x)\ \{ Fx \rightarrow (\forall y)\ (f(x, y) \rightarrow Fy) \}$

If from the proposition that δ has the property *F*, whatever δ may be, it can be inferred that every result of an application of the procedure *f* on δ has the property *F*, so I say:

> 'the property *F* **inherits** in the *f*-series′ (*Bs*., §24).

We write this as *HP(F,f)* to emphasise the dependence on *F* and on *f*.

- Definition of "following in a series"



(76)

If from both propositions, that from each result of the application of the procedure *f* on *x* has the property *F*, and that the property *F* inherits in the *f*-series, whatever *F* maybe, it can be inferred that *y* has the property *F*, then I say:

> "*y* follows *x* in the *f*-series " or
> "*x* precedes *y* in the *f*-series" (*Bs*., §26).

(SUC) $(\forall F)\ \{(\mathrm{HP(F,f)}\ \&\ (\forall z\ (f\,(a, z) \rightarrow Fz)) \rightarrow Fb\}$ signifies "*b* follows *a* in the *f*-series".

II – Kerry´s criticism

-"Über Anschauung und ihre psychische Verarbeitung", IV, pp. 294–295:

> So liegt aber die Sache (...) bei der den eigentlichen Gegenstand unserer Betrachtung ausmachenden des Folgens von *y* auf *x* in der *f*-Reihe[1]). Dieselbe hat genau erwogen den Sinn, dass *y* als auf *x* in der *f*-Reihe folgend dann bezeichnet werden solle, wenn darauf geschlossen werden kann, dass *y* alle sich in der *f*-Reihe vererbenden Eigenschaften besitze. Nun ist dieses Kriterium schon darum von zweifelhaftem Werthe, weil kein Katalog solcher Eigenschaften existirt, man also nie sicher ist, den Inbegriff derselben erschöpft zu haben. Hiezu kommt aber als ausschlaggebend noch der Umstand, dass, wie unser Autor selbst nachgewiesen hat[2]), eine der in der *f*-Reihe sich vererbenden Eigenschaften auch ist: in der *f*-Reihe auf *x* zu folgen. Hienach hängt die Entscheidung darüber, ob *y* auf *x* in der *f*-Reihe folge, laut der für diesen Begriff gegebenen Definition davon ab, dass man, nebst sehr vielem Anderen über vererbende Eigenschaften überhaupt, speciell von der vererbenden Eigenschaft: auf *x* zu folgen, Das wisse, ob *y* sie besitze oder nicht (Kerry IV, pp. 294–295).
>
> [1]) Vgl. oben, S. 270, 293.
> [2]) Begriffsschrift, S. 71 (Formel 97).

- Summarizing, Kerry points out:

i) the impossibility of precisely determining the set of hereditary properties,
ii) that "following *x*" itself is one of such hereditary properties.

III – Russell´s primary views on Frege and Kerry

- 1903 *The Principles of Mathematics*, “Appendix A”, p 522.

> The definition of immediate sequence in the series of natural numbers is also severely criticized (p. 292 ff.). This depends upon the general theory of series set forth in Bs. Kerry objects that Frege has defined “*F* is inherited in the *f*-series,” but has not defined “the *f*-series” nor “*F* is inherited”. The latter essentially ought not to be defined, having no precise sense, the former is easily defined, if necessary, as the field of the relation *f*”.

- 1906 “On some difficulties in the theory of transfinite numbers and order types”, *Proc. London Math. Soc.*, Ser. 2, vol. 4, parte I, 7.3.1906, 29-53.

- 1908 “Mathematical Logic as Based on the Theory of Types”.

- 1910 Whitehead y Russell, *Principia Mathematica*, vol. I.

IV – <u>Concluding remarks</u>

i) <u>Kerry *1887*</u> is an historical antecedent of **Poincaré *1906*** in his denial of those kind of definitions which, also in 1906 a little bit earlier, during a talk in the London Mathematical Society Russell would call "non-predicative enunciative forms".

ii) In Kerry *1887* there´s a historical and of eventual influence of the Principle of Vicious Circle (Russell, 1908, "Mathematical Logic as Based on the Theory of Types"). Cf. Russell *1903* and Linsky *2005*.

iii) If Fregean logicism is to be taken as an epistemological project, as Frege himself explicitly states in, e. g., his sharp philosophical work, (1884) *Die Grundlagen der Arithmetik*, then it fails from the very beginning, due to the circularity of the definition of succession.

Some primary bibliography

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