G = omega-categorical group
H = infinite subgroup of G .
H is called **inert** in G if for every g from G the intersection $H \cap gHg^{-1}$ is of finite index in H .

Proposition 1 . Every finite subgroup of G is contained in an infinite residually finite inert subgroup of G.

G is **residually finite** if every finite subset F of non-trivial elements of G has empty intersection with some normal subgroup of G of finite index.

Proposition 1 follows from the main results of [**V.Belyaev**, Locally finite groups with a finite non-separable subgroup, S.Math.J., 1993];
$H<G$ is **separable**, if there is a non-trvl finite H-invariant $K<G$ with $H\cap K=1$.

Let H be an inert residually finite subgroup of G and let
$X = \{ gK : K \cap H$ is of finite index both in K and $H \}$.

By [Belyaev] the action of G on X by multiplication defines an embedding of G into $Sym(X)$ such that the closure G^c is locally compact and the closure H^c is a compact subgroup of G^c .

If G is residually finite the completion G^c with respect to G is the profinite completion of G .

Proposition 2. G^c is locally finite.
If G is abelian (k-step nilpotent, soluble), then G^c is abelian (k-step nilpotent, soluble resp.) .

Proposition 3.
G is finite-by-abelian-by-finite if and only if there is a positive real $\varepsilon<1$ such that for any infinite compact $C<G^c$ we have

$$\mu_C(\{(x,y)\in C\times C : xy=yx\})=\varepsilon,$$

where μ_C is the normalized Haar measure on C .

A group K has countable (topological) **cofinality** if K can be presented as the union of an ω-chain of proper (open) subgroups.
What is the cofinality of G^c ?

Observation. If G^c is not compact then G^c has countable topological cofinality.

Apps-Wilson Theorem.
An ω-categorical group G has a finite series $1=G_0<G_1 < \ldots <G_n = G$ with each G_i characteristic in G, and with each G_i /G_{i-1} either elementary abelian, or isomorphic to some Boolean power P^R with finite simple non-abelian P and atomless Boolean ring R , or an ω-categorical characteristically simple non-abelian p-group for some p.

Theorem 1. The completion G^c has uncountable cofinality if and only if G is residually finite and the following properties hold:

1. In any series $1=G_0<G_1 < \ldots<G_n=G$ with each G_i/G_{i-1} characteristically simple the highest infinite quotient G_m/G_{m-1} (with maximal m) is isomorphic to some Boolean power P^R with finite simple non-abelian P ;

2. For any abelian cover $N_1 <N_2$ in the lattice of characteristic subgroups of G the $G^c/N^c{}_2$-module $N^c{}_2 /N^c{}_1$ does not have countable $G^c/N^c{}_2$-cofinality .

Proposition 4. If the group G is locally soluble, then G^c has countable cofinality.

Theorem 2.
Let G be residually finite and let both G and G^c be ω-categorical. Then if G is soluble then both G and G^c are nilpotent-by-finite.

Observation. Under the assumptions above let $N_1 < N_2 < G$ be a cover in the lattice of characteristic subgroups of G. If G^c is ω-categorical and the closures N^c_1 and N^c_2 are definable in G^c then N^c_2 / N^c_1 is abelian.

Proposition 5. If G is residually finite and G^c satisfies the same AE-sentences with G then there is a number m such that for any g from G^c any element of the minimal normal subgroup of G^c containing g is a product of $< m$ conjugates of g
(G^c has finite conjugate spread).