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Abstracts of

Plenary Talks

Tutorials

Special Sessions

Contributed Talks

Contents

Plenary Talks

▸ MIKLÓS AJTAI, *First-order definable bijections and the notion of cardinality.*
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The set theoretical definition that two sets have the same cardinality involves the existence of a bijection between them. In a more algorithmic setting it is more natural if, in the definition of cardinality, we restrict our attention to bijections which are given in some constructive way. We investigate this question in the case when the sets are subsets of $\{0, 1\}^n$, for a natural number n, which are the sets of zeros of a constant depth polynomial size unlimited fan-in boolean circuit (AC_0 circuit), or equivalently subsets of the set $\{1, ..., n\}$ which can be defined by a first-order formula on a suitable structure whose universe is $\{1, ..., n\}$. We restrict our attention to bijections computable in both direction by an AC_0-circuit, or equivalently bijections where, in both directions, each bit of the image can be defined by a first-order formula on a suitable structure.

We show that the notion of cardinality in this constructive world is essentially different from the set theoretical one, namely there are sets with identical cardinalities in the set theoretical sense so that there is no AC_0 computable bijection between them in either directions. The talk will also include results about AC_0-definability which are needed for the proof.

▸ AKIHIRO KANAMORI, *Bernays and set theory.*
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30 years after his passing, we take this opportunity to survey the work of Paul Bernays and its influence on modern set theory.

▸ ROMAN, KOSSAK, *Automorphisms of models of PA: the neglected cases.*
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Automorphism groups of countable recursively saturated models of PA were studied intensively in the 1990's. In a series of papers (joint and separate) by Lascar, Kaye, Kotlarski, Schmerl and the author, the important notion of arithmetic saturation was isolated and a series of results characterizing arithmetic saturation in group theoretic terms were proved. The notions involved include: generic automorphisms, maximal automorphisms, the small index property, maximal open subgroups, and the cofinality of the group. Most of this material is presented in [1]. Much less has been done concerning models which are recursively saturated and not arithmetically saturated (we do not even have a good name for this class of models), and even less concerning short recursively saturated models. I will give a survey of this entire area of research and I will include some more recent results concerning the neglected cases, in particular the problem of extendabability of automorphisms to cofinal extensions. This last topic is joint work with Henryk Kotlarski.

[1] Roman Kossak, James Schmerl, ***The Structure of Models of Peano Arithmetic***, Oxford Logic Guides 50, Oxford University Press, 2006.

▸ HANNES LEITGEB, *On Truth and Probability.*
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We will give an overview of how probability theory can be studied from a logical point of view. On the probabilistic side, this leads us to an investigation of (unary) absolute probability measures and (binary) conditional probability measures on propositional or first-order languages. In particular, we will show (i) how the standard axioms of probability may be justified in terms of "closeness to the truth", (ii) how logical truth can be pinned down by purely probabilistic means, (iii) how conditional probability measures on formulas may be represented as limits of ratios of absolute probability measures on formulas, (iv) what a probabilistic semantics for conditional logic looks like, (v) how a probabilistic theory of truth for semantically closed languages can be developed, and finally (vi) how models for higher-order probabilities, probabilistic reflection principles, and type-free probability can be constructed.

(Most parts of this talk will be based on unpublished or forthcoming material.)

► AMADOR MARTIN-PIZARRO, *Categoricity and the classification of strongly minimal sets.*
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Strongly minimal sets, i.e. irreducible sets of dimension one, were first identified as the principal component of any uncountably categorical structure by Michael Morley in his proof of Los' uncountable categoricity conjecture. They were subsequently recognized to be the main building block of any structure of finite Morley rank; Boris Zilber conjectured that they fall essentially into one of three well-known types. This was refuted by an ingenious construction of Hrushovski, which is the principal tool of creating exotic strongly minimal sets. The talk will be mostly oriented to an audience not familiar with Model Theory, based on examples to illustrate the results exhibited.

► JOSEPH S. MILLER, *Extracting information is hard.*
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Can randomness – or more technically, "information" – be effectively extracted from a semi-random source? A special case of this question was answered by von Neumann in 1951. He described a simple method for extracting an unbiased random sequence from the flips of a biased coin. A more general form of the question was asked by Reimann and Terwijn in the context of algorithmic randomness, so we will start with an introduction to Kolmogorov complexity, effective Hausdorff dimension, and Martin-Löf randomness. Kolmogorov complexity measures the information content of a finite binary string. Informally, the complexity of a string is defined to be the length of the shortest program that generates it. A closely related notion, effective (Hausdorff) dimension, measures the information density of an infinite binary sequences. We can now formulate the question in terms of effective dimension: is there a sequence that has effective Hausdorff dimension $\frac{1}{2}$ – in other words, a half-random sequence – that does not compute a sequence of higher effective dimension? As it turns out, such sequences exist. We will discuss this and related results.

► JAAP VAN OOSTEN, *Geometric Aspects of the Effective Topos.*
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The effective topos, discovered by Martin Hyland in 1979, is a generalization to full higher-order type theory of Kleene's realizability interpretation of 1945. It is one coherent mathematical universe in which also recursive analysis, in the words of Hyland, "finds its natural home". Moreover, various classically inconsistent theories which did not originate in recursive interpretations of mathematics have been shown to admit models in the effective topos, such as synthetic domain theory, algebraically compact categories and Aczel's Constructive Set Theory (augmented with several nonclassical axioms).

In this talk, we shall explore to what extent constructions known from abstract homotopy theory can be performed in the effective topos, and which properties of the category of simplicial sets find analogues in it.

► THOMAS SCANLON, *Algebraic dynamics from the model theory of difference fields.*
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In algebraic dynamics, one studies the behavior iterated applications of rational functions while with the model theory of difference fields concerns the structure of definable sets in existentially closed difference fields. I will discuss recent work in which problems in algebraic dynamics have been expressed in terms of the model theory of difference fields and thereby resolved or at least clarified.

► STEPHEN G. SIMPSON, *Mass Problems.*
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Kolmogorov 1923 proposed to view intuitionistic logic as a "calculus of problems" (Aufgabenrechnung). This is essentially the famous BHK interpretation of intuitionism. Medvedev 1955 introduced mass problems as a rigorous elaboration of Kolmogorov's proposal. A *mass problem* is a set of reals. If P is a mass problem, the *solutions* of P are the elements of P. We say that P is *solvable* if there exists a computable solution of P. We say that P is *weakly reducible* to Q if each solution of Q can be used as a Turing oracle to compute some solution of P. A *weak degree* is an equivalence class of mass problems under mutual weak reducibility. Let $\mathcal{D}_w$ be the lattice of weak degrees. There is an obvious, natural embedding of the Turing degrees into $\mathcal{D}_w$, obtained by identifying the Turing degree of a real with the weak degree of the singleton set consisting of that real. Muchnik 1963 observed that $\mathcal{D}_w$ is a model of intuitionistic propositional calculus. Since 1999 I have been studying the sublattice $\mathcal{P}_w$ consisting of the weak degrees of nonempty effectively closed sets in Euclidean space. I have discovered that there is a natural embedding of the recursively enumerable Turing degrees into $\mathcal{P}_w$. Moreover, I have discovered that $\mathcal{P}_w$ contains a variety of specific, natural, weak degrees which are closely related to various foundationally interesting topics. Among these topics are reverse mathematics, algorithmic randomness, algorithmic information theory, hyperarithmeticity, diagonal nonrecursiveness, almost everywhere domination, subrecursive hierarchies, resource-bounded computational complexity, effective Hausdorff dimension, and Kolmogorov complexity. Recently I have applied $\mathcal{P}_w$ to the study of 2-dimensional symbolic dynamics. The purpose of this talk is to introduce $\mathcal{P}_w$ and to survey what is known about it.

► LAJOS SOUKUP, *Rainbow Colourings.*
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Given a function $f : [X]^n \to X$ a subset Y of X is a *rainbow subset for f* provided $f \restriction [Y]^n$ is one-to-one. A typical question is the following old problem of Erdős: *Assume that the colouring $f : [\omega_1]^2 \to 3$ establishes the negative partition relation $\omega_1 \not\to [\omega_1]^2_3$. Is there a rainbow triangle for f?*

We survey some old and new results and we also present some new theorems. E.g we show that if a colouring c establishes $\omega_2 \not\to [(\omega_1;\omega)]^2_\omega$ then c establishes this negative partition relation in each Cohen-generic extension of the ground model, i.e. this property of c is Cohen-indestructible. This result yields a negative answer to a question of Erdős and Hajnal: it is consistent that GCH holds and there is a colouring $c : [\omega_2]^2 \to 2$ establishing $\omega_2 \not\to [(\omega_1;\omega)]^2_2$ such that some colouring $g : [\omega_1]^2 \to 2$ can not be embedded into c.

It is also consistent that 2^{ω_1} is arbitrarily large, and there is a function g establishing $2^{\omega_1} \not\to [(\omega_1,\omega_2)]^2_{\omega_1}$ but there is no uncountable g-rainbow subset of 2^{ω_1}.

We also show that if GCH holds then for each $k \in \omega$ there is a k-bounded colouring $f : [\omega_1]^2 \to \omega_1$ and there are two c.c.c posets $\mathcal{P}$ and $\mathcal{Q}$ such that

$$V^{\mathcal{P}} \models \text{“}f \text{ c.c.c-indestructibly establishes } \omega_1 \not\to^* [(\omega_1;\omega_1)]_{k-bdd}\text{,”}$$

but

$$V^{\mathcal{Q}} \models \text{“ } \omega_1 \text{ is the union of countably many } f\text{-rainbow sets. ”}$$

▶ THOMAS STRAHM, *Weak theories of operations and types.*

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The theories of operations and types addressed in this talk are formulated in the spirit of Feferman's explicit mathematics. It has turned out that this framework provides a natural axiomatic basis for studying notions of abstract computability, especially from a proof-theoretic perspective. Our emphasis in this talk is on proof-theoretically weak systems and their relationship to complexity classes and weak arithmetics.

In the first part of our talk, we are concerned with so-called type-free applicative theories, which form the operational core of systems of explicit mathematics. In an applicative universe of discourse, all objects may be thought of as operations or rules, which can freely be applied to each other: self-application is meaningful, though not necessarily total. We survey results on various weak applicative theories whose provably total operations coincide with the functions computable in polynomial time, linear space, and polynomial space. Our systems can be seen as natural applicative analogues of systems of bounded arithmetic.

It is a distinguished advantage of our applicative theories that they allow for a very intrinsic and direct discussion of higher type issues, since higher types arise naturally in our combinatory setting. Moreover, due to the fact that the untyped language does not a priori restrict the class of functionals which can be expressed, it makes perfect sense to consider the class of higher type functionals which are provably total in a given applicative system. We will present some proof-theoretic aspects of higher type complexities in our setting; in particular, we will address the Melhorn-Cook-Urquhart basic feasible functionals.

The second part of the talk will be devoted to extensions of the first-order applicative framework by various means of typing disciplines. We study weak explicit typing and naming in the spirit of explicit mathematics. In particular, we discuss weak type systems with a restricted form of elementary comprehension whose provably terminating operations coincide with the polytime functions.

An interesting alternative of extending the applicative core by classes or types is by means of a partial (self-referential) truth predicate. We discuss results of Cantini who has studied substantial extensions of weak first-order applicative theories by partial truth, an axiom of choice and a positive uniformity principle.

► MATTEO VIALE, *Forcing axioms, supercompact cardinals, singular cardinal combinatorics.*

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I will give a survey on my work over the combinatorics at the successor of a singular cardinal in these two different scenarios:

- The singular cardinal is larger than a strongly compact cardinal κ and of cofinality smaller than κ,
- A strong forcing axiom like Martins maximum or the proper forcing axiom PFA holds.

Under these assumptions I will show that a variety of combinatorial objects which I call covering matrices are useful tools to attack many of the classical problems on successor of singular cardinals, for example we can obtain proofs of Solovay's theorem that the singular cardinal hypothesis SCH holds above a strongly compact cardinal, or that PFA implies SCH, or a recent characterization by me and Assaf Sharon of the structure of the approachability ideal for successors of singular cardinals of the type described above.

Tutorials

▸ ANAND PILLAY, *Compact spaces and definability.*
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This tutorial will hopefully have some appeal to both a general audience of logicians as well as to researchers in model theory. The aim is to describe various interactions between definability in model theory and compact spaces, with reference to current research.

In the introductory first lecture I will discuss the category of definable sets (for a given first order theory), as well as notions such as hyperdefinability. I will also introduce various notions of strong type and state a conjecture relating the Lascar group of a first order theory to the complexity of Borel equivalence relations.

In the second lecture I will discuss Keisler measures, or probability measures on type spaces, and give certain consequences, for theories without the independence property, of the Vapnis-Chervonenkis theorem (uniform law of large numbers).

In the third lecture I will introduce the notion of a definable set (or group) being dominated by a compact space (or group), and describe some recent results.

The second and third lectures are related to joint work with Ehud Hrushovski, as well as being closely related to Peterzil's tutorial in LC '07.

▸ MICHAEL RATHJEN,
Infinitary proof theory and its uses.
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Proof theory was the technical area of logic instituted by Hilbert in the early 20th century in order to carry out his programme: to lay to rest all worries about the foundations of mathematics once and for all by securing mathematics via an absolute proof of consistency. Strong restrictions were placed on the methods to be applied in consistency proofs of axiom systems for mathematics: namely, these methods were to be completely *finitistic* in character. Hilberts Programme is a reductive enterprise with the aim of showing that whenever a real proposition can be proved by ideal means, it can also be proved by real, finitistic means. However, Hilbert's so-called formalism was not intended to eliminate nonconstructive existence proofs in the practice of mathematics, but to vindicate them.

In the 1920s, Ackermann and von Neumann, in pursuit of Hilberts Programme, were working on consistency proofs for arithmetical systems, but already with the infusion of an infinitary concept. Ackermann's 1924 dissertation gives a consistency proof for a second-order version of primitive recursive arithmetic which explicitly uses a finitistic version of transfinite induction up to the ordinal $\omega^{\omega^{\omega}}$. The employment of transfinite induction on ordinals in consistency proofs came explicitly to the fore in Gentzen's 1936 consistency proof for Peano arithmetic. His proof became the paradigm for an *ordinal analysis* of a theory by assigning a *proof-theoretic ordinal* to a theory which calibrates its logical power.

Beginning in the 1950s, proof theory began to employ infinitary methods more frankly, first with the use of infinitary rules of inference (such as the omega rule) and then with the use of infinitely long formulas. At first, these rules, formulas and the resulting derivations were all countable, but eventually proof theory moved on to make use of prima facie uncountably long rules of inference and/or formulas, thence uncountably long derivations. These countably and uncountably long derivations have been applied to treat various formal systems for parts of analysis and set theory, by translating their finite derivations into corresponding infinite ones of a more purely logical character. By then applying the process of cut-elimination or other forms of normalization, the latter are transformed into detour-free derivations whose associated ordinals give a measure of complexity to the systems thus treated. For example, they can be used to classify their provably recursive functions. Cut-free derivations are easily shown not to end in a contradiction, so one also concludes consistency from such analyses. The more that such infinitary methods were employed, the closer did proof theory come to ongoing developments in recursion theory, particularly as generalized to admissible sets; in both one makes use of analogues of regular cardinals, as well as "large" cardinals (inaccessible, Mahlo, etc.).

The lectures will trace the remarkable transformation of proof theory up to developments in recent years. Notwithstanding that it employs infinitary derivations, proof theory provides finitistic reductions between theories, especially Π^0_2 conservation results. Moreover, the lectures will explore its applications in logic, combinatorics, computer science and constructivism. If time permits, we will also take up the question of where the boundaries of current ordinal-theoretic proof theory lie and what are the main obstacles to be overcome.

▸ STEVO TODORCEVIC, *Combinatorial Dichotomies in Set Theory*.
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Early investigations of set-theoretic Forcing Axioms have led us to several dichotomies of combinatorial nature that are easy to understand and use. This will be a survey of some of these dichotomies with a special emphasis to their applications. We shall also expose the influence of these dichotomies on the cardinality of the continuum and its other cardinal invariants.

Special Sessions

Computability and Arithmetic

▸ BARBARA F. CSIMA, *Degree spectra of almost computable structures*.
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A countable algebraic structure is almost computable if almost every Turing degree can compute a copy of the structure; in other words, if the degree spectrum of the structure has measure 1 under the standard measure on the Cantor space. We give examples of almost computable structures the complements of degree spectra of which are uncountable. Moreover, we give an example of a structure whose degree spectrum coincides with the hyperimmune degrees.

▸ ANTONIN KUCERA, *Properties of PA sets and random sets*.
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We discuss methods of coding information into PA-sets and ML-random sets (e.g. how to make a given set T-reducible to some PA-set or ML-random set). Slaman and Kucera used these techniques for PA-sets to provide a characterization of ideals in the T-degrees for which there is a low T-upper bound. As a corollary, there is a PA-set which is a low T-upper bound for the class of K-trivial sets. In the case of ML-random sets, the situation is naturally more complicated and the techniques are less powerful. We survey some results in this area including new developments, e.g. a result of Barmpalias and Montalban that any K-trivial set is T-reducible to an incomplete (even low) ML-random set.

▸ SHAHRAM MOHSENIPOUR, *A Bezout Computable Nonstandard Model of Open Induction*.
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In contrast with Tennenbaum's theorem [8] that says Peano Arithmetic (PA) has no nonstandard computable model, Shepherdson [6] constructed a computable nonstandard model for a very weak fragment of PA, called Open Induction (Iopen) in which the induction scheme is only allowed to be applied for quantifier-free formulas (with parameters). Since then several attempts have been made from both sides to strengthen Tennenbaum's and Shepherdson's theorems. From one direction one would like to find fragments of arithmetic as weak as possible with no nonstandard computable model. On the other hand we are also interested in knowing those fragments that are as strong as possible and do have a computable nonstandard model. Attempts in the first direction were culminated in the work of Wilmers [10] where it is shown that IE_1 does not have a computable nonstandard model (IE_1 is the fragment based on the induction scheme for bounded existential formulas). Our work deals with the second direction. Since Open Induction is too weak to prove many true statements of number theory (It cannot even prove irrationality of $\sqrt{2}$), a number of algebraic first order properties have been suggested to be added to Iopen in order to obtain closer systems to number theory. These properties include: Normality [2], having the GCD property [7], being a Bezout domain [3], cofinality of primes (abbreviated here as cof(prime)) and so on. We mention that GCD is stronger than normality, Bezout is stronger than GCD and Bezout is weaker than IE_1. Berarducci and Otero [1], based on earlier works of Wilkie

[9], van den Dries [2] and Macintyre-Marker [3] constructed a computable nonstandard model for Iopen + Normality + cof(prime). Also Moniri [5] by using transseries, managed to generalize Shepherdson's method directly, to construct primitive recursive nonstandard models of Iopen + cof(prime) with any finite transcendence degree > 1. In [4] we succeeded to strengthen Berarducci-Otero's construction by combining their method with that of Smith [7](which is itself a generalization of Macintyre-Marker's work to the GCD and Bezout case) and obtained a nonstandard computable model of Iopen+GCD+cof(prime). In this talk, we go one step further by bringing all of these materials together (Smith's chains, Berarducci-Otero's computable construction and Moniri's transseries) to produce a computable nonstandard model of Open Induction which is Bezout and has cofinal primes.

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[6] Shepherdson, J. C., *A non-standard model for a free variable fragment of number theory,* Bull. Acad. Polon. Sci. **12** (1964) 79-86.

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▸ NEIL THAPEN, *The provably total search problems of bounded arithmetic.*
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We give natural combinatorial principles GI_k, based on sequences of k-turn games, which are complete for the class of NP search problems provably total at the kth level T_2^k of the bounded arithmetic hierarchy and hence characterize the $\forall\Sigma_1^b$ consequences of T_2^k. Our argument uses a translation of first order proofs into large, uniform propositional proofs in a system in which the soundness of the rules can be witnessed by polynomial time reductions between games.

Logic and Computer Science

▸ MARIANGIOLA DEZANI-CIANCAGLINI, *Logical Semantics.*
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Stone dualities allow to describe special classes of topological spaces by means of (possibly finitary) partial orders. Typically, these partial orders are given by the topology, a basis for it, or a subbasis for it. The seminal result is the duality between the categories of Stone spaces and that of Boolean algebras (see [23]). Other very important examples are the descriptions of *Scott domains* as *information systems* [27] and the description of *SFP domains* as *pre-locales* [1]. It is worthwhile to mention also Martin-Löf's domain interpretation of intuitionistic type theory [24].

Intersection types can be viewed also as a restriction of the domain theory in logical form, see [1], to the special case of modelling pure lambda calculus by means of ω-algebraic complete lattices. Intersection types have been used as a powerful tool both for the analysis and the synthesis of λ-models, see *e.g.* [3], [8], [2], [17], [16], [25], [21], [26]. On the one hand, intersection type disciplines provide finitary inductive definitions of interpretation of λ-terms in models. On the other hand, they are suggestive for the shape the domain model has to have in order to exhibit certain properties, [9], [22], [13], [14], [28], [15].

More recently intersection (together with union) types have been used to build *fully abstract* models of extensions of the λ-calculus including parallel features [4], [11], [12], [5], of Higher-Order Processes [19], [20], [18], of the π-calculus [10] and of ambient calculi [7], [6].

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▸ KAZUSHIGE TERUI, *Towards a logical foundation of computational complexity.*
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Traditional theories of computational complexity are built on concrete machine models. As a result, the nature of explanation often tends to be procedural (eg. via transformation of machines). Aiming at a more algebraic account to complexity theoretic phenomena (eg. via logical isomorphisms), we propose a new foundation based on logic and functional computation (in the paradigm of Curry-Howard correspondence).

The underlying framework is ludics of J.-Y. Girard, that is a pre-logical system upon which ordinary logics are to be built and analyzed. Due to its monistic nature, data and machines are both represented by abstract proofs, and the notion of acceptance is replaced by a fundamental notion of orthogonality. Space efficient composition of algorithms (which is usually explained by a peculiar construction of machines) is very naturally achieved by adopting a pointer-based normalization procedure (known as

Krivine's abstract machine).

After introducing the basic formalism, we shall illustrate how some basic complexity theoretic phenomena (eg. space compression) are explained by logical properties (eg. focalization) and logical isomorphisms.

▸ YDE VENEMA, *Logic and automata: a coalgebraic perspective.*
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A long and respectable tradition in theoretical computer science, going back to the work of Büchi and Rabin, links the research fields of automata theory and logic. This link becomes particularly strong for automata operating on (potentially) *infinite* objects like streams, trees or transition systems. An interesting phenomenon in this area is that most (but not all) key results hold for word and tree automata alike, and that many can even be formulated and proved for automata that operate on yet other objects such as trees of unbounded branching degree, or labelled transition systems. The aim of the talk is to show that these results can be formulated and proved at the more general abstraction level of *coalgebra*.

Universal Coalgebra is an emerging mathematical theory of state-based evolving systems. Words, trees and transition systems are all examples of coalgebras of a certain type, which is formally given as a functor F on the category Set (with sets as objects and functions as arrows). We introduce the concept of an F-automata, a device that operates on coalgebras of type F. The criterion under which such an automaton accepts or rejects a pointed coalgebra is formulated in terms of an infinite two-player graph game. Extending Moss' coalgebraic logic with fixpoint operators, we also introduce a language of coalgebraic fixed point logic for coalgebras, and we provide a game semantics for this language.

Finally we show that some of the central *results* in automata theory can be generalized to the abstraction level of coalgebras and thus lay out the foundations of a universal theory of automata. As examples of such results, we will see that the class of recognizable languages of coalgebras is closed under taking unions, intersections, and projections. We also prove that if a coalgebra automaton accepts some coalgebra it accepts a finite one of bounded size. Many of these results are based on an explicit construction which transforms a given alternating F-automaton into an equivalent nondeterministic one, of bounded size.

[1] C. Kupke and Y. Venema, *Coalgebraic automata theory: basic results*, ***Logical Methods in Computer Science***, to appear.

[2] Y. Venema, *Automata and fixed point logic: a coalgebraic perspective*, ***Information and Computation***, 204:637–678, 2006.

▸ TING ZHANG, *Knuth-Bendix Order and Its Decidability.*
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Several kinds of orderings, such as polynomial orderings, lexicographic path orderings and Knuth-Bendix orderings, are widely used in term rewriting and theorem proving. In term rewriting, these orderings are powerful tools in proving termination of rewriting systems. In theorem proving, they are key instruments to prune search space without compromising refutational completeness. Solving ordering constraints is therefore essential to the successful application of ordered rewriting and ordered resolution. Besides the needs for decision procedures for existential theories, situations arise in constrained deduction where the satisfiability of arbitrarily quantified formulas need be decided.

Unfortunately, the first-order theory of lexicographic path orderings is undecidable, so is existential theory of polynomial orderings. This leaves an open question whether the first-order theory of Knuth-Bendix orderings is decidable (RTA problem 99). In this talk, we give a positive answer to this question using quantifier elimination on a complex structure containing term algebras and integer arithmetic. In fact, we shall show the decidability of a theory that is more expressive than the theory of Knuth-Bendix orderings.

Model Theory

▸ MARTIN HILS, *Generalised Hrushovski Constructions and Generic Automorphisms*.
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Hrushovski's amalgamation method is a powerful tool to construct (finite rank) structures with interesting combinatorial geometries. Ehud Hrushovski first used it in [2] to construct a counter-example to Zilber's Trichotomy Conjecture, then to fuse two strongly minimal theories into a single one. The method later served to construct various unexpected expansions of algebraically closed fields, most recently a *bad field* in characteristic 0 [1]. It is useful to divide the method into two steps: the *free amalgamation*, where a theory T_ω (usually of infinite rank) is obtained; then the *collapse* of T_ω onto the desired finite rank theory.

We will focus on generalisations of the free amalgamation method, in particular the free fusion of two simple rank 1 theories *over a common ω-categorical reduct*. In interesting cases, the resulting theory is supersimple [3]. The situation is similar to the expansion of some (stable) theory by a *generic automorphism*.

In important algebraic contexts, e.g. for algebraically closed fields, the class of models together with a generic automorphism is axiomatisable. We show that the same is true for some of the infinite rank structures obtained by Hrushovski's free amalgamation method.

[1] A. Baudisch, M. Hils, A. Martin Pizarro and F. O. Wagner: *Die böse Farbe*, to appear in J. Inst. Math. Jussieu.

[2] E. Hrushovski, *A new strongly minimal set*, Ann. Pure Appl. Logic **62** (1993), 147–166.

[3] M. Hils: *La fusion libre : le cas simple*, to appear in Proceedings of the conference 'Logicum Lugdunensis' (special volume of the J. Inst. Math. Jussieu).

▸ GARETH JONES, *Model completeness and o-minimality*.
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Establishing model completeness is one of the main methods for showing that an expansion of the real field is o-minimal. There are also results in the other direction, showing that certain o-minimal structures are model complete (in a reasonable language). Previous results of this type only applied to restricted functions. I shall discuss a result which applies to unrestricted functions, provided that the structure is polynomially bounded. I shall also give examples and applications.

▸ MOSHE KAMENSKY, *Internality to algebraically and differentially closed fields*.
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I will review the model theoretic notion of internality, and the associated Galois theory. I will then review some applications in the particular context of internality to theories of fields.

▸ KRZYSZTOF KRUPINSKI, *Some model theory of Polish structures.*
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I have introduced Polish structures in order to apply model theoretic ideas in the studies of purely descriptive set theoretic and topological objects such as Polish G-spaces or, more generally, Borel G-spaces. In particular, Polish structures generalize profinite structures introduced by Newelski. Polish structures allow us to apply ideas and techniques from model theory, descriptive set theory, topology and the theory of profinite groups.

A Polish structure is a pair (X, G) where G is a Polish group acting (faithfully) on a set X so that the stabilizers of all points are closed subgroups of G. We say that (X, G) is small if for every natural number n there are countably many orbits on X^n under G.

A simple non-profinite example of a small Polish structure is the unit circle with the full group of homeomorphisms. In fact, most natural examples of compact metric spaces with the full group of homeomorphisms are small Polish structures. More complicated examples are Hilbert cube and the pseudo-arc with the full group of homeomorphisms.

I will discuss a purely topological notion of independence, called non-meager independence, that satisfies some nice properties (e.g. symmetry, transitivity, existence of independent extensions) in small Polish structures, and so allows us to introduce basic stability-theoretic concepts and to prove fundamental results about them (e.g. Lascar inequalities). This notion of independence coincides with m-independence introduced by Newelski in profinite structures.

In the second part of my talk I will concentrate on the structure of small compact G-groups, i.e. small Polish structures (X, G) where X is a compact group and G acts continuously on X as a group of automorphisms. I will present an example of such a group which is not solvable-by-finite. On the other hand, under a natural model theoretic assumption of 'superstability' with respect to nm-independence, each such group is solvable-by-finite, and assuming finiteness of the underlying rank, it is even nilpotent-by-finite.

I will finish with some open questions.

Set Theory

▸ PANDELIS DODOS, *Descriptive Set Theory and the Geometry of Banach spaces.*
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We shall discuss some recent advances on the interaction between Banach Space Theory and Descriptive Set Theory. We will focus on classical problems in the Geometry of Banach spaces (problems going back to the beginnings of the theory) which are of the following type:

> Suppose that (P) is a certain property of Banach spaces. Suppose further that we are given a class C of separable Banach spaces such that every space in the class C has property (P). When can we find a separable Banach space Y which has property (P) and contains an isomorphic copy of every member of the given class C?

As it turns out, for most natural properties, an affirmative answer to the above problem is possible if and only if the class C is "simple". The "simplicity" of the class is measured in set theoretic terms. In particular, if the class C is analytic in a natural coding of separable Banach spaces, then we can indeed find a space Y as required above.

We will present the historical background and overview related results, starting from the seminal work of Jean Bourgain in the 80s which led, eventually, in the solution the last few years.

► TAMÁS MÁTRAI, *On a new construction of σ-ideals of compact sets.*
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We present a new construction of $\mathbf{\Pi}^0_2$ σ-ideals of compact sets. To shed light on the role of this construction, we take a tour of the recent developments of the theory of σ-ideals of compact sets. We are mainly concerned with open and recently solved problems related to regularity properties and Tuckey reducibility.

► KATHERINE THOMPSON, *Obtaining inner models for global results using class forcing.*
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The forcing technique to produce outer models has been exploited to a great extent to show an enormous range of possibilities which may be true of the universe. Relative to a fixed universe, some of the same results can be true in inner and outer models, but different techniques are used. S. Friedman introduced a programme which aims to discover what questions may be answered in an inner model.

There are many consistency results achieved using forcing which are not yet known to be obtainable via inner models, since internal consistency results are harder to achieve. To obtain any consistency result, one must show that a generic filter exists. To achieve internal consistency, one needs an inner model with all the ordinals of the ambient universe, therefore one cannot restrict to a countable submodel, as is usual in set-forcing. The question then is how to build generics, if one even exists.

One class of results which are studied in this context are called "global properties". These are statements which are true throughout the universe (e.g. at every regular cardinal). Global properties are achieved using class forcing. This adds an extra challenge to building generics as all antichains of the forcing which exist in the universe must be considered.

In this talk, we will examine the techniques used to build such generics and the properties of the forcings which are sufficient to utilise these methods.

► TODOR TSANKOV, *Positive definite functions on equivalence relations.*
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We consider positive definite functions on countable equivalence relations, in particular ones arising from subequivalence relations. We discuss various applications, including ones to a recent co-inducing construction of Epstein and orbit equivalence as well as percolation on Cayley graphs of property (T) groups. This is joint work with A. Ioana and A. Kechris.

Contributed Talks

▸ BAHAREH AFSHARI AND MICHAEL RATHJEN, *Theories of Iterated Positive Induction.*

In 1963, G. Kreisel [5] initiated the study of formal theories featuring inductive definitions. Subsystems of the theories of iterated inductive definitions (ID_n) such as the fixed point theories $\hat{ID}_n$ where investigated by Aczel and Feferman in connection with Hancock's conjecture about the strength of Martin-Löf type theories with universes. Another interesting type of theory lying between $\hat{ID}_n$ and the usual ID_n is ID^*_n. To illustrate this in the case $n = 1$, in contrast to $\hat{ID}_1$, ID^*_1 has an induction principle for the fixed points but it is restricted to formulas in which other fixed points occur only positively. Results about the theories ID^*_n were obtained by Friedman, Feferman [4], and Cantini [2]. However, they did not settle the proof-theoretic strength of the theories ID^*_n. I would like to talk about our recent results revealing the strength of these theories.

[1] Wilfried Buchholz, Solomon Feferman, Wolfram Pohlers and Wilfried Sieg. *Iterated inductive definitions and subsystems of Analysis: Recent Proof-Theoretical Studies theories*, Springer-Verlag, Berlin, Heidelberg, 1981.

[2] Andrea Cantini. A note on a predicatively reducible theory of elementary iterated induction, *Bollettino U.M.I.*, pp. 413-430, 1985.

[3] Andrea Cantini. On the relation between choice and comprehension principles in second order arithmetic, *Journal of Symbolic Logic* **51**, pp. 360–373, 1986.

[4] Solomon Feferman. Iterated inductive fixed-point theories: Application to Hancock's conjecture, *Patras Logic Symposion*, pp. 171–196, North-Holland, Amsterdam, 1982.

[5] G. Kreisel. Generalized inductive definitions. Tech. rep., Stanford University, 1963.

[6] Michael Rathjen. Auwahl und Komprehension in Teitsystemen der Analysis, M.Sc. thesis, University of Münster, Germany, 1985.

[7] K. Schütte. *Proof Theory*, Springer-Verlag, Berlin, Heidelberg, 1977.

[8] Helmut Schwichtenberg. Proof Theory: Some Applications of Cut-Elimination, *Handbook of Mathematical Logic*, pp. 868–895, North-Holland, 1977.

[9] Stephen G. Simpson. *Subsystems of Second Order Arithmetic*, Springer-Verlag, Berlin, Heidelberg, 1999.

▸ PAVEL ALAEV, *Ideals in computable Boolean algebras with distinguished ideals.*
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We consider computable Boolean algebras with a finite number of distinguished ideals (I-algebras), i.e., structures of the form $(\mathfrak{A}^*, I_1, \dots, I_\lambda)$, where $\mathfrak{A}^*$ is a computable Boolean algebra and $I_1, \dots, I_\lambda$ are computable ideals in $\mathfrak{A}^*$. A relation R is said to be intrinsically computable (c.e., ...) if for every isomorphism $f : \mathfrak{A} \to \mathfrak{B}$ onto a computable structure $\mathfrak{B}$ the image $f(R)$ is computable (c.e., ...), and to be relatively intrinsically computable if for every isomorphism $f : \mathfrak{A} \to \mathfrak{B}$ onto an arbitrary structure $\mathfrak{B}$ with elements in ω the image $f(R)$ is computable with respect to the diagram $D(\mathfrak{B})$.

We consider the problem of describing intrinsically computable ideals in $\mathfrak{A}$. This problem can be considered as related to investigating decidable theories. A complete algebraic description is found for relatively intrinsically computable, c.e. and co-c.e. ideals in $\mathfrak{A}$, and also for intrinsically computable ideals in the case $\lambda \leqslant 2$.

Let the set $L_1, \dots, L_m$ denote all finite intersections $\bigcap_{t \in T} I_t$, $\{0\}$ and $\mathfrak{A}$. Let $R \subseteq \mathfrak{A}$

be an ideal. We say that R is a Σ-ideal if $R = \sum_{s\in S} L_s$ or $\{0\}$; R is a Π-ideal, if $R = \bigcap_{(t,s)\in X}(L_t \to L_s) \cap L_n$, and R is a Δ-ideal if R is a Σ-ideal and a Π-ideal simultaneously. Here $L_1 + L_2 = \{x + y \mid x \in L_1, y \in L_2\}$, $L_1 \to L_2 = \{x \in \mathfrak{A} \mid \forall y \leqslant x (y \in L_1 \to y \in L_2)\}$, $a_1, \dots, a_n \mid 1$ means that $a_1 + \ldots + a_n = 1$ and $a_i \cdot a_j = 0$ for $i \neq j$, and $\widehat{a}$ means the restriction of $\mathfrak{A}$ to the set $\{x \in \mathfrak{A} \mid x \leqslant a\}$.

Theorem 1. An ideal R is relatively intrinsically c.e. in $\mathfrak{A}$ if and only if there exists a set $a_1, \dots, a_n \mid 1$ such that $R \cap \widehat{a}_i$ is a Σ-ideal in $\widehat{a}_i$ for all $i \leqslant n$.

Theorem 2. An ideal R is relatively intrinsically co-c.e. in $\mathfrak{A}$ if and only if there exists a set $a_1, \dots, a_n \mid 1$ such that for each $i \leqslant n$ at least one of the following holds:
(a) $R \cap \widehat{a}_i$ is a Π-ideal in $\widehat{a}_i$;
(b) $R \cap \widehat{a}_i$ is a Σ-ideal and a maximal ideal in $\widehat{a}_i$ simultaneously.

Corollary. An ideal R is relatively intrinsically computable in $\mathfrak{A}$ if and only if there exists a set $a_1, \dots, a_n \mid 1$ such that for each $i \leqslant n$ at least one of the following holds:
(a) $R \cap \widehat{a}_i$ is a Δ-ideal in $\widehat{a}_i$;
(b) $R \cap \widehat{a}_i$ is a Σ-ideal and a maximal ideal in $\widehat{a}_i$ simultaneously.

Theorem 3. Let $\mathfrak{A} = (\mathfrak{A}^*, I_1, I_2)$ be a computable I-algebra with two distinguished ideals. An ideal R is intrinsically computable in $\mathfrak{A}$ if and only if R is relatively intrinsically computable.

▸ LUCA ALBERUCCI, *Sequent Calculi for the Modal μ-Calculus over* $\mathsf{S5}$.
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In the joint work with Facchini [1] the author shows that fixpoints of the modal μ-calculus over $\mathsf{S5}$ are reached after two iterations by using game theoretical methods.

In the first part of our talk we present a sequent calculus with analytic cut for the modal μ-calculus over $\mathsf{S5}$ where the fixpoint rules reflect the fact that they are reached after two iterations. Using a classical canonical countermodel construction we prove completeness. Correctness follows from the result mentionned at the beginning.

In the second part we present a calculus with usual fixpoints rules. In this case correctness follows from standard arguments. In order to show completeness we prove, by using purely syntactical methods, that a fixpoint formula is provably equivalent to its second stage approximation. As a corollary we get a completeness result for Kozen's Axiomatisation $+\mathsf{S5}$ axioms for the modal μ-calculus over $\mathsf{S5}$ without using the already known completeness of Kozen's Axiomatisation over K.

[1] Luca Alberucci and Alessandro Facchini: *The Modal μ-Calculus Hierarchy over Restricted Classes of Transition Systems*. Submitted.

▸ MICHAEL ARNDT, *Logical Tomography*.
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Gentzen's formulation of the sequent calculus draws heavily on the purely structural logic of Paul Hertz. Having been strongly opposed to what he termed "logical conventionalism", the mistaking of the manipulation of logical language for reasoning, Hertz restricted his investigations on sets of clauses of the form $a_1, \dots, a_m \to b$, expressing the dependency of an element b on the set of elements $a_1, \dots, a_m$. Reasoning in this structural logic corresponds to the generation of new clauses by means of multicut.

By allowing a more general form of clauses $a_1, \dots, a_m \to b_1, \dots, b_n$, it is possible to relate any classical propositional Gentzen-style sequent to a uniqe set of such purely structural clauses by a simple procedure using the cut rule of LK and local variants of its logical rules. Moreover, a suitable generalisation of Hertz' graph-like illustrations of sets of clauses results in a succinct representation of these sets by means of cut-hypergraphs.

[1] Gerhard Gentzen, *Über die Existenz unabhängiger Axiomensysteme zu unendlichen Satzsystemen*, ***Mathematische Annalen***, vol. 107 (1933), pp. 329–350.

[2] ——— *Untersuchungen über das logische Schließen. I-II*, ***Mathematische Zeitschrift***, vol. 39 (1934/35), pp. 176–210, 405–431.

[3] Paul Hertz, *Über Axiomensysteme für beliebige Satzsysteme. I. Teil. Sätze ersten Grades* ***Mathematische Annalen***, vol. 87 (1922), pp. 246–269.

[4] ——— *Über Axiomensysteme für beliebige Satzsysteme. II. Teil. Sätze höheren Grades*, ***Mathematische Annalen***, vol. 89 (1923), pp. 76–100.

[5] ——— *Über Axiomensysteme für beliebige Satzsysteme*, ***Mathematische Annalen***, vol. 101 (1929), pp. 457–514.

[6] ——— *Sprache und Logik*, ***Erkenntnis***, vol. 7 (1937), no. 1, pp. 457–514.

[7] Peter Schroeder-Heister, *Resolution and the origins of structural reasoning: Early proof-theoretic ideas of Hertz and Gentzen* ***The Bulletin of Symbolic Logic***, vol. 8 (2002), pp. 246–265.

▸ MATTHIAS BAAZ, STEFAN HETZL, *On the non-confluence of cut-elimination.*
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Cut-elimination is an inherently non-deterministic process. At each stage one has the choice between different cuts to reduce and – for a single cut – there are different ways of reducing it. It is however not clear, in how far this formal non-determinism may lead to mathematical differences in the resulting elementary proofs.

Understanding the possible span of the results of cut-elimination procedures is of fundamental importance for judging proof analyses based on these methods, like for example Girard's demonstration that from the topological Fürstenberg-Weiss proof of van der Waerden's theorem, the original combinatorial proof can be obtained.

In this talk we study the standard syntactic cut-elimination procedure for first-order classical logic. We show that the process of generating the Herbrand-universe of a given term signature can be encoded in a proof with cuts: Its cut-elimination will non-deterministically compute a term out of all terms of a certain size. Moreover, the proof with cuts is free of weakening (which would otherwise permit trivial non-confluent configurations).

Based on this construction we define a sequence of short proofs exhibiting a strongly non-confluent cut-elimination behaviour: A proof in this sequence has non-elementarily many different cut-free normal forms (each of non-elementary size). These normal forms are different in a strong sense: They not only represent different Herbrand-disjunctions but also have different propositional structures and hence are not substitution-instances of a single general proof.

This result shows that the constructive content of a cut-free proof does not only depend on the proof with cuts from which it was generated but also from the way the cuts are eliminated.

▸ BENNO VAN DEN BERG, *Realizability in algebraic set theory.*
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The aim of algebraic set theory [2] is to provide a uniform categorical semantics for set theories of different kinds (classical or constructive, predicative or impredicative, well-founded or non-well-founded, etc.). In this talk we show how realizability methods for constructive set theories like **IZF** and **CZF** fit into the framework of algebraic set theory. This allows us to simultaneously recover known realizability interpretations of Friedman, McCarty and Rathjen, and introduce some unfamiliar ones. (The contents of this talk are based on [1], which is the second in a series of papers on algebraic set theory, written together with Ieke Moerdijk.)

[1] B. van den Berg & I. Moerdijk, *Aspects of predicative algebraic set theory II: realizability*, arXiv:0801.2305, submitted for publication.

[2] A. Joyal & I. Moerdijk, ***Algebraic set theory***, London Mathematical Society Lecture Note Series, volume 220, Cambridge University Press, 1995.

▸ ULRICH BERGER, *A coinductive definition of uniform continuity and its application to program extraction from proofs*.

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We give a coinductive definition of the set of (uniformly) continuous real functions on a compact interval and extract from various proofs programs that construct and combine exact real number algorithms with respect to the binary signed digit representation of real numbers.

For program extraction we use an extension of Kreisel's modified realisability interpretation that has some similarity with the interpretation by Tatsuta [5]. The interpretation assigns to the coinductive definition of continuous functions a data type of finitely branching non-wellfounded trees describing when an algorithm for a continuous function writes and reads digits.

A noteworthy feature of the extracted programs is the fact that, when executed in a lazy programming language, previous computations are memoized. This can result in a considerable performance improvement if a function is evaluated at a large number of inputs that are close together

Coinductive definitions of real numbers and exact real number computation with respect to (generalised) digits have been studied by e.g. in [1, 2, 4]. New in our approach is that not only real numbers, but continuous functions are represented coinductively and programs computing them are extracted from coinductive proofs.

[1] Yves Bertot, *Affine functions and series with co-inductive real numbers*, ***Mathematical Structures in Computer Science***, vol. 17 (2007), no. 1, pp. 37–63.

[2] Alberto Ciaffaglione and Pietro di Gianantonio, *Affine functions and series with co-inductive real numbers*, ***Theoretical Computer Science***, vol. 351 (2006), no. 1, pp. 39–51.

[3] Abbas Edalat and Reinhold Heckmann, *Computing with real numbers - I. The LFT approach to real number computation - II. A domain framework for computational geometry*, ***International summer school on applied semantics,*** (Gille Barthe, Peter Dybjer, Luis Pinto, Saraiva, J., editors), Springer, Caminha, Portugal, Berlin, 2002, pp. 193–267.

[4] Michal Konečný, *Real functions incrementally computable by finite automata*, ***Theoretical Computer Science***, vol. 315 (2004), no. 1, pp. 39–51.

[5] Makoto Tatsuta, *Realizability of monotone coinductive definitions and its application to program synthesis*, ***Proceedings of the Fourth International Conference on Mathematics of Program Construction, Lecture Notes in Computer***

Science, vol. 1422, Springer, 1998, pp. 338–364.

▶ MARTA BÍLKOVÁ, *On complexity of ML and MTL*.
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We consider one of open problems in complexity of weak fuzzy logics - that of complexity of Monoidal t-norm based logic MTL [2]. We do not solve the problem but rather relate it to other open problems in complexity of substructural logics - namely to the complexity of provability from finite theories in Höhle's monoidal logic ML (which is also known as Full Lambek calculus with exchange and weakening $\mathrm{FL}_{\mathsf{ew}}$ or intuitionistic multiplicative aditive linear logic with weakening IMALLW), and to the complexity of intuitionistic linear logic with weakening ILLW.

We reduce the provability in MTL to provability with finite theories in ML (only known to be decidable [1]), and to provability in ILLW. While for ML without weakening rule complexity is known (IMALL is known to be PSPACE-complete [3], with finite theories undecidable as full LL), for ML with finite theories, ILL and ILLW it is as far as we know open. We present a game for provability in ML (with and without theories) and reason about its proof theoretical and complexity flavour.

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▶ ACHIM BLUMENSATH, BRUNO COURCELLE, *Interpretation degrees for guarded second-order logic*.
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We study guarded second-order interpretations between classes of finite structures. The main result is a complete description of the resulting hierarchy. It is linear of order type $\omega+3$. Each level can be characterised in terms of a suitable variant of tree-width. Canonical representatives of the various levels are: the class of (i) all trees of height n, for $n<\omega$; (ii) all paths; (iii) all trees; and (iv) all grids.

▶ GABRIELA CAMPERO-ARENA, JOHN K. TRUSS, *Countable 1-transitive cyclic orderings*.
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We give a classification of all the countable 1-transitive coloured cyclic orderings as in [3]. We use the equivalent definitions of cyclic ordering given in [4] and [6], and understand a colored cyclic ordering as a cyclic ordering in which each point is assigned a member of a set C, thought of as its 'colour'. By '1-transitivity' we mean that the automorphism group acts singly transitively on each set of points coloured by a fixed colour. As any cyclic ordering may be built from a linear ordering, we use the classification of the countable 1-transitive coloured linear orderings, given in [1] and [2] to achieve the classification of the corresponding cyclic orderings. We prove that the two methods given in [6] for constructing a cyclic ordering from a linear one preserve 1-transitivity, and that the more involved method is necessary to get all the 1-transitive cyclic orderings considered.

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We investigate computable subshifts. Π^0_1 subshifts are constructed in $2^{\mathbb{N}}$ that have different and incomparable Medvedev degrees.

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► AN. CHUBARYAN, ARM. CHUBARYAN, *On the proof complexities in the Frege systems with different substitution rules.*
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We compare the proof complexities in Frege systems with multiple substitution rule and with constant bounded substitution rule. We use the generally accepted concept of Frege system $\mathcal{F}$ as follows: it uses a finite complete set of propositional connectives and it has a finite set of schematically defined rules of inference; furthermore it must be sound and complete.

A substitution Frege system $S\mathcal{F}$ consists of a Frege system $\mathcal{F}$ augmented with the substitution rule with inferences of the form $\frac{A}{A\sigma}$ for any substitution σ, consisting of a mapping from propositional variables to propositional formulas, and $A\sigma$ denotes the result of applying the substitution to formula A, which replaces each variable in A with its image under σ. This definition of substitution rule allows to use the simultaneous substitution of multiple formulas for multiple variables of A without any restrictions.

If for any constant integer $k \geqslant 1$ we allow substitution for only no more than k variables at a time, then we have k-bounded substitution rule. The k-bounded substitution Frege system $S_k\mathcal{F}$ consists of a Frege system $\mathcal{F}$ augmented with the k-bounded substitution rule.

We use also the well-known notions of proof complexities (size or steps) and the notion of polynomially equivalence (by size or by steps) of the proof systems. We prove that

1) for every fixed integers k_1 and k_2 the systems $S_{k_1}\mathcal{F}$ and $S_{k_2}\mathcal{F}$ are polynomially equivalent (both by size and by steps);
2) for every fixed integer k the systems $S_k\mathcal{F}$ and $S\mathcal{F}$ are polynomially equivalent by size;
3) there is a sequence of tautologies φ_n of the length $\theta(n)$ such that for every fixed integer k the number of steps needed to prove φ_n in $S_k\mathcal{F}$ is $\Omega(n)$, but it can be proved using only $O(\log_2 n)$ steps in $S\mathcal{F}$.

▸ ANNALISA CONVERSANO, *Definable polar decomposition of o-minimal groups*.
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Many authors in the last twenty years have obtained results that show a very strong connection between definable groups in o-minimal structures and real Lie groups. However an important difference is represented by the existence of compact subgroups.
It is well-known that every connected real Lie group has a polar decomposition, instead in the o-minimal context there are examples of definably connected definable groups not simply connected but without any infinite definably compact definable subgroups.
We'll prove that every definable group G in an o-minimal expansion of a real closed field R contains a definably contractible definable subgroup H such that the quotient G/H has a definable polar decomposition, namely it contains a definably compact definable subgroup K and a definable subspace X definably homeomorphic to R^n such that the product of G/H gives a definable homeomorphism $K \times X \cong G/H$. Moreover G is definably homotopically equivalent to G/H and so we'll deduce properties of G from K.
We will also show that in general G/G^{00} is a normal subgroup of K/K^{00}, where G^{00} is the smallest type-definable subgroup of bounded index in G and G/G^{00} has the logic topology. If G is solvable or G/H is definably compact then $G/G^{00} = K/K^{00}$.

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The alternative constituent format displays linguistic or logical data in a succinct way that permits ready grasp of subtle contrasts. A grammatical constituent of a sentence is replaced by a sequence of alternatives. Consider the ambiguous question Q.

(Q) Is two closer to three than four?

Consider an initial sentence A as an affirmative answer.

(A) Two is closer to three than four.

We can compare this with a negative answer by replacing the constituent is with the choice between is and is not.

(AN) Two (is * is not) closer to three than four.

We can also compare the initial ambiguous sentence A with two other unambiguous sentences that give different readings or interpretations. Replace the constituent four with the choice among three alternatives: four, to four, and four is. Asterisks separate choices.

(1) Two is closer to three than (four * to four * four is).

The first alternative yields the initial, elliptically and structurally ambiguous sentence; the second and third alternatives give two of its readings (second true, third false). This presents three sentences in one alternative constituent string or ACS. Two constituents can be varied independently.

(2) (Zero * One* Two) is closer to three than (four * to four * four is).

Here, nine sentences are presented in one ACS: the three with four are ambiguous; the three with to four are true; and the three with four is are false. Any number of constituents can be varied. A constituent can be zeroed using a blank as in the following.

(AN1) Two is (* not) closer to three than four.

This yields the same two ambiguous alternatives as ANemphasizing the relation of the choice to affirm and the choice not to deny.

▸ LAURA CROSILLA, *Constructive predicativity.*
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The notion of predicativity took shape in the writings of Poincaré and Russell in response to the paradoxes of set theory. It subsequently saw the fundamental contribution of Weyl.

Different notions of predicativity have been proposed over the years, and also distinct logical analysis of them have emerged. The most accredited analysis is due to Feferman and Schütte (independently) following the lines indicated by Kreisel. This analysis makes use of proof–theoretic techniques and singles out an ordinal, Γ_0, which is seen as constituting a limit for the proof–theoretic strength of a predicative theory.

It is usually agreed that constructive mathematics should adhere to a form of predicativity. By constructive mathematics we here denote the variety of constructive mathematics known as Bishop–style mathematics, after the seminal work of its founder E. Bishop.

For constructive theories, however, Feferman and Schütte's bound has been seen as too restrictive. Some authors have suggested that *inductive definitions* should enter the realm of constructive mathematics. This has the effect of extending the reach of constructive predicativity. In fact, classical studies on the proof theoretic strength of theories of inductive definitions have shown that they go well beyond Feferman and Schütte's bound.

Though the admissibility of some form of inductive definitions within the constructive practice appears now to be undisputed, a thorough analysis of the resulting notion is still lacking.

In the context of Martin–Löf type theory, for example, the upper limits of predicativity have been recently approached by constructions which use universes (sometimes in conjunction with so–called W–types). An analysis of these constructions reveals that non–monotone inductive definitions are required.

We here present a contribution towards an analysis of constructive predicativity.

▸ JUSTUS DILLER, *Functional interpretations of classical systems.*

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Classical theories can (and should) be formalized without mention of existential quantification and disjunction. Then, as Peano Arithmetic PA is Dialectica interpreted in Gödels's theory T of primitive recursive functionals, its neutral finite type version PA^{ω} is Diller-Nahm interpreted in $T^c_{\wedge}$, an extension of T by a bounded universal quantifier. Also Kripke-Platek set theory $KP\omega$ is Diller-Nahm interpreted in Burr's bounded theory $T^c_{\in}$ of constructive set functionals. It is only for the interpretation of Aczel's Constructive Set Theory that we need a new translation of the existential quantifier.

▸ NATASHA DOBRINEN, SY-DAVID FRIEDMAN, *Homogeneous iteration and measure one covering relative to HOD*.

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Assuming that there is no inner model with a Woodin cardinal, Steel constructs a certain inner model, K^c, from which the 'true' core model K for a Woodin cardinal is obtained. An important lemma in the derivation of K from K^c is the following, due to Steel: If κ is a measurable cardinal with a normal measure μ, then $\{\alpha < \kappa : \alpha^+ = (\alpha^+)^{K^c}\}$ has μ-measure one. This is called measure one covering relative to K^c [1].

Since the inner model K^c is contained in HOD, the universe of hereditarily ordinal definable sets, it is natural to ask whether measure one covering might fail relative to HOD. Indeed, we prove that, relative to a hyperstrong cardinal, it is consistent that measure one covering fails relative to HOD; that is, it is consistent that there is a measurable cardinal κ with a normal measure μ such that the set $\{\alpha < \kappa : \alpha^+ = (\alpha^+)^{\mathrm{HOD}}\}$ has μ-measure zero. Along the way, we prove a preservation theorem for iterations of homogeneous forcings [2].

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▸ DAMIR D. DZHAFAROV AND CARL G. JOCKUSCH, JR., *Ramsey's theorem and cone avoidance*.

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The computability theoretic content of Ramsey's Theorem in combinatorics has been the subject of much study. Jockusch [3] asked whether every computable 2-coloring of pairs admits a homogeneous set which does not compute $0'$, and whether every computable 2-coloring of pairs admits a low_2 homogeneous set.

Seetapun [4] affirmatively answered the first by showing that for any sequence $C_0, C_1, \ldots$ of noncomputable sets, every computable 2-coloring of pairs admits a homogeneous set H with $C_i \not\leq_T H$ for all i. Cholak, Jockusch, and Slaman [1] affirmatively answered the second question. In turn, they asked whether for any given noncomputable set C, every computable 2-coloring of pairs admits a low_2 homogeneous set H with $C \not\leq_T H$.

In this paper, we obtain an affirmative answer to this question by two different methods. The first combines control of the first jump with a new proof of Seetapun's result which is considerably more straightforward than both the original and the subsequent simplification of it due to Hummel and Jockusch [2]. The other controls the second jump directly and yields the following stronger theorem: every computable 2-coloring of pairs admits a pair of low_2 homogeneous sets whose degrees form a minimal pair.

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▸ PHILIP EHRLICH, *Theories of Continua*.
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Paul du Bois-Reymond [5], Giuseppe Veronese [8], and Charles Sanders Peirce [see 4 for references] all proposed theories of continua that made use of infinitesimals. Du Bois-Reymonds and Veroneses theories were given precision by Hausdorff [6] and Levi-Civita [7], respectively, and in [4] we provided a formal replacement for the purported continuum of Peirce. In [2], [3] and [4] we have suggested that whereas the standard continuum $\mathbb{R}$ of real numbers should be regarded as an arithmetic continuum modulo the Archimedean axiom, Conways ordered field No of surreal numbers may be regarded as a sort of absolute arithmetic modulo NBG. Here we show how the (aforementioned formalizations of the) theories of du Bois-Reymond, Veronese, and Peirce along with $\mathbb{R}$ are naturally situated in No.

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▸ RACHEL EPSTEIN, *Prime models of computably enumerable degree.*
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We examine the computably enumerable (c.e.) degrees of prime models of complete atomic decidable (CAD) theories. Every CAD theory has a prime model, but not necessarily a computable prime model. Previous research has dealt with the Δ_2 degrees of the elementary diagrams of prime models, but the c.e. degrees remained unexplored. We show that many results for the Δ_2 degrees can be strengthened to hold for the c.e. degrees. The results for Δ_2 degrees used finite extension oracle constructions, while for the c.e. case, we use infinite injury arguments. Let $D^e(\mathcal{M})$ denote the elementary diagram of the model $\mathcal{M}$. We show that every CAD theory T has prime models $\mathcal{M}$ and $\mathcal{M}'$ such that the degrees of $D^e(\mathcal{M})$ and $D^e(\mathcal{M}')$ form a minimal pair of low c.e. degrees. This strengthens Csima's result that every CAD theory has a minimal pair of low prime models. Next we prove that if a CAD theory T has a prime model $\mathcal{M}$ such that $D^e(\mathcal{M})$ has c.e. degree $\mathbf{c}$, then we can push down the degree to get a prime model $\mathcal{M}'$ of T such that $D^e(\mathcal{M}')$ has c.e. degree $\mathbf{b} < \mathbf{c}$. This pushdown theorem holds for the c.e. degrees but not for the Δ_2 degrees where $\mathbf{c}$ may have minimal degree. We also give a density theorem which states that for any c.e. degrees $\mathbf{d} < \mathbf{c}$ where $\mathbf{c}$ is not low$_2$ ($\mathbf{c}'' > \mathbf{0}''$), then every CAD theory has a prime model $\mathcal{M}$ such that $D^e(\mathcal{M})$ is between $\mathbf{d}$ and $\mathbf{c}$ and has the lowest possible jump degree $\mathbf{d}'$.

▸ ALESSANDRO FACCHINI AND LUCA ALBERUCCI, *The Modal μ-Calculus Hierarchy over Restricted Classes of Transition Systems.*
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In this talk we discuss the strictness of the modal μ-calculus hierarchy on some restricted classes of transition systems. First, we show that the hierarchy is strict over reflexive frames and, by proving the finite model theorem for reflexive systems, the same results holds on finite models. Then, we prove that the hierarchy collapses to the alternation-free fragment over transitive systems. This is done by giving an explicit syntactical translation of the full modal μ-calculus into the alternation-free fragment which preserves logical equivalence for finite systems. By proving a finite model theorem this translation can be extended to all transition systems. Moreover, we verify that if symmetricity is added to transitivity, the hierarchy collapses to the purely modal fragment. In this case this is done by giving an explicit syntactical translation of the full modal μ-calculus into the modal fragment, too. The results of this talk are presented in [1].

[1] Luca Alberucci and Alessandro Facchini: *The Modal μ-Calculus Hierarchy over Restricted Classes of Transition Systems*. Submitted.

▸ HENRIK FORSSELL, *First-order logical duality.*
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Stone duality can be seen as a 'syntax-semantics' duality for propositional logic by regarding Boolean algebras as Lindenbaum-Tarski algebras of propositional theories and the corresponding Stone spaces of ultrafilters as spaces of models. Representing the syntax of first-order theories by Boolean coherent categories, we generalize Stone duality to first-order logic by constructing a duality between Boolean coherent categories and certain 'semantical' topological groupoids. The heart of the construction is a representation of the topos of coherent sheaves on a Boolean coherent category as equivariant sheaves on the topological groupoid of set-valued coherent functors and invertible natural transformations; a representation which allows us to recover the syntax of a first-order theory from its models and model-isomorphisms. Compared to the well-known duality between Boolean pretopoi and ultragroupoids constructed by Michael Makkai [1], this construction is more geometrical in that it uses topology and sheaves rather than structure based on ultraproducts. Moreover, it specializes to the classical Stone duality for Boolean algebras.

[1] M. Makkai, ***Duality and definability in first order logic***, Memoirs of the American Mathematical Society 503, 1993.

▸ GUNTER FUCHS, JOEL HAMKINS, JONAS REITZ, *Set Theoretic Geology.*
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We investigate some new kinds of inner models. The basic idea is to invert forcing: Instead of looking at forcing extensions of the universe, we look at grounds of the universe. A ground is an inner model of which the universe is a forcing extension. We refer to the the intersection of all grounds of V as the mantle. The generic mantle of V is the intersection of all grounds of all set forcing extensions of V. We show that the generic mantle is always an inner model of Zermelo-Fraenkel set theory, and we prove that every model of ZFC is the mantle and generic mantle of another model of ZFC. Other kinds of inner models result from iterating the process of forming mantles, which gives the inner mantles, and from intersecting the HODs of generic extensions, which gives what we call the generic HOD. The latter is a ZFC model. Many fundamental questions touching upon the basics of forcing remain open.

▸ MARA GABRIELA FULUGONIO, *Gottlob Frege: Begriffsschrift and Impredicativity.*
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In the spirit of the 19th mathematical project of arithmetization analysis, Frege aims still a further goal: to demonstrate that arithmetic can be properly expressed by pure logic concepts. Thus, in 1879 shows up the Begriffsschrift and with it the very first complete first-order calculus with identity axiomatization. Although the importance of the Begriffsschrift in the history of logic is quite difficult to be exaggerated, it is well known that it remained mostly underappreciated, when even not understood at all, and that this annoyed Frege deeply. Even though, Frege himself seems to ignore that he had in the young Austrian Benno Kerry (1858-1889) the sharpest reader of the days. Nowadays, we know of Benno Kerry, basically, as the one who inspired (1892) On Concept and Object. But just going through the eight articles conforming the series that Frege quotes in his famous 1892 article reveals we are in front of an authentic philosopher, very well informed on the 19th centurys problematic on fundaments of mathematics. In particular, Kerry goes into Freges work in his II and III articles and accuses his logicist reduction of the induction principle of being circular.

In my work I aim to show that such critics is an antecedent of the (today known as) impredicativity problem and that it is also an antecedent of Russells Vitiosus Circulus Principle, which was formulated in 1908 as an eventual justification of his type theory. With such a purpose on mind, I offer in I an analysis of the fregean definition of succession; in II, I present Kerrys critics and, in III, Russells comments about it in his Appendix A of (1903) Principles of Mathematics. I leave IV for conclusions.

▸ RICHARD GARNER, *Two-dimensional models of Martin-Löf type theory.*
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It has been known for a long time that a reasonable notion of categorical model for Martin-Löf's extensional dependent type theory [2] is that of a locally cartesian closed category [1]. Yet much less is known about the correct categorical models for intensional Martin-Löf type theory [3]. Whilst it is quite easy to write down categorical structures which capture the logic faithfully, such structures tend to look more like transliterations of the type-theoretic syntax into categorical language than they do like a genuinely semantic notion of categorical model.

Recently, a number of authors (following a suggestion of I. Moerdijk) have drawn attention to parallels between intensional Martin-Löf type theory and certain aspects of homotopy theory and higher-dimensional category theory [4, 5]. Thus it is natural to seek models of intensional type theory which are themselves higher-dimensional categories. The purpose of this talk is to describe the first few steps along this road.

We describe a type theory which we call *two-dimensional type theory*, and which lies somewhere between the intensional and extensional versions of Martin-Löf type theory; and we then sketch a correspondence between two-dimensional type theories and a class of two-dimensional categorical models. Finally, we explain how any intensional type theory gives rise to a two-dimensional model through an internal groupoid construction.

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▸ PHILIPP GERHARDY, *Proof mining in topological dynamics.*
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"Proof mining" is the activity of extracting additional data from proofs in mathematics and computer science. Proof mining roughly falls into two parts: on the one hand, proving general metatheorems that classify theorems and proofs from which one can extract interesting additional information, on the other hand, carrying out case studies. In this talk, we will focus on the latter discussing two topological proofs of van der Waerden's Theorem and their computational content.

van der Waerden's Theorem ([3]) states that for any finite colouring of the integers one colour contains arbitrarily long arithmetic progressions, where an arithmetic progression of length k is a sequence $a, a+b, a+2b, \ldots, a+(k-1)b$ for integers a, b.

Equivalently, for integers q, k there is an N such that any q-colouring of $[0, N]$ admits a monochromatic progression of length k. One may then ask for the growth rate of N in q and k.

The bounds given by van der Waerden in his original combinatorial proof are of Ackermann complexity, while one may show an exponential lower bound on $N(q, k)$. There is a theorem in topological dynamics equivalent to van der Waerden's theorem, the Multiple Birkhoff Recurrence Theorem. We analyse two different proofs of the Multiple Birkhoff Recurrence Theorem, the original one by Furstenberg and Weiss ([1]) and a variant by Girard ([2]), and discuss the relationship between the bounds extracted from these proofs and van der Waerden's original combinatorial treatment. A paper in which the results of this analysis appear has been submitted.

[1] H. Furstenberg, B. Weiss, *Topological Dynamics and Combinatorial Number Theory*, ***Journal d'Analyse Mathematique***, vol.34(1978), pp. 61–85.

[2] J.-Y. Girard, ***Proof Theory and Logical Complexity. Volume I***, Bibliopolis, Naples, 1987.

[3] B.L. van der Waerden, *Beweis einer Baudetschen Vermutung*, ***Nieuw Archief voor Wiskunde***, vol. 15(1927), pp. 212–216.

▸ KAVEH GHASEMLOO, *Complexity theory over domains*.
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Domain Theory is one of approaches toward foundations for Computable Analysis [1, 4]. A weak point of it in comparison to other models, e.g. [5] is lack of a Complexity Theory. The reason behind this problem is that Domain Theory's main objective was to model computability abstractly, where as for complexity we need more information about the process of computation, which a Domain does not provide, i.e. a Domain does not provide how good an approximation is w.r.t. total elements (*quality of approximation*). We solve this by adding a Scott-continuous function q from a Domain D to $([0, 1], \leq)$. Continuity is plausible. Total elements are those with $q(x) = 1$. The choice of q can vary, depending on what is important for the application. This is a generalization of the situation occurring in Solid Domains[2], where two measures for quality of Solid Objects are provided, depending on the application. We show that our model generalizes [2] approach to arbitrary domains, and for a suitable q, is compatible with Wierauch's approach in [5], and therefore with Ko's [3] approach.

[1] Abbas Edalat, *Domains For Computaion In Mathematics, Physics And Exact Real Arithmetic*, ***Bulletin of Symbolic Logic***, vol. 3(4), (1997), pp. 401-452.

[2] Abbas Edalat and André Lieutier, *Foundation of a Computable Solid Modelling*, ***Proceedings of the fifth ACM symposium on Solid Modeling and applications*** ACM, 1999, pp. 278–284.

[3] Ker-I Ko, ***Complexity Theory of Real Functions***, Birkhäuser, 1991.

[4] Viggo Stoltenberg-Hansen and John Tucker, *Computability on Topological Spaces via Domain Representations*, ***New Computational Paradigms: Changing Conceptions of What is Computable*** (S. Barry Cooper and Benedikt Löwe and Andrea Sorbi, editors), Springer, Publisher's address, 2007, pp. 153–194.

[5] Klaus Weirauch, ***Computable Analysis, An Introduction***, Texts in Theorical Computer Science, Springer, 2000.

▸ ZUZANA HANIKOVÁ AND PETR SAVICKÝ, *On standard SBL-algebras with added involutive negations*.
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Basic (Fuzzy) Logic BL is a propositional calculus introduced by Hájek in [3]. In [1], it was shown to be complete w. r. t. the interpretation by continuous t-norms and their residua on $[0,1]$ (its standard semantics).

Strict Basic Logic SBL extends BL with an axiom guaranteeing that the BL-definable negation is the strict (two-valued) negation. [2] introduces SBL and investigates its expansions with another negation, which is involutive.

Given a continuous SBL t-norm, we investigate the problem whether adding non-isomorphic involutive negations gives rise to distinct sets of propositional tautologies. Among continuous SBL t-norms which are finite ordinal sums of Lukasiewicz (Ł) and product (Π) components, we arrive at the following full characterization of those with the above property: (1) If the ordinal sum is Π, $\Pi \oplus j.\text{Ł}$, or $\Pi \oplus i.\text{Ł} \oplus \Pi \oplus j.\text{Ł}$, for $i \geq 0$, $j > 0$, then any two non-isomorphic involutive negations determine distinct and incomparable sets of propositional tautologies. (2) Otherwise (if the sum is of type $\Pi \oplus i.\text{Ł} \oplus \Pi$ or it contains at least three product components), one can construct two non-isomorphic negations which—together with the given t-norm—determine the same set of propositional tautologies.

[1] Roberto Cignoli, Francesc Esteva, Lluís Godo, and Antoni Torrens, *Basic Fuzzy Logic is the logic of continuous t-norms and their residua*, ***Soft Computing***, vol. 4, pp. 106–112, 2000.

[2] Francesc Esteva, Lluís Godo, Petr Hájek, and Mirko Navara, *Residuated fuzzy logic with an involutive negation*, ***Archive for Mathematica Logic***, vol. 39, pp. 103–124, 2000.

[3] Petr Hájek, ***Metamathematics of Fuzzy Logic***, Trends in Logic – Studia Logica Library, Kluwer Academic Publishers, 1998.

▸ VALENTINA HARIZANOV, RUSSELL MILLER, AND ANDREI MOROZOV, *Automorphism spectra and tree-definability*.

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The *automorphism spectrum* of a computable structure $\mathcal{A}$ is the set of Turing degrees of nontrivial automorphisms of $\mathcal{A}$. Thus it measures the complexity of the symmetries of $\mathcal{A}$. We present several results about which sets of Turing degrees can be automorphism spectra of computable structures, and in particular, for which single degrees $\boldsymbol{d}$ a computable structure can have automorphism spectrum $\{\boldsymbol{d}\}$. This condition is equivalent to $\boldsymbol{d}$ being *tree-definable*, i.e. to the existence of a computable subtree $T \subseteq \omega^{<\omega}$ such that there is exactly one path p through T and $\deg(p) = \boldsymbol{d}$.

▸ CHRIS IMPENS, SAM SANDERS[†], *Dangerous axioms of nonstandard arithmetic*.

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In Foundations of Mathematics, understanding theories of arithmetic can be greatly enhanced by studying their nonstandard extensions. The usual nonstandard tools (Transfer, Saturation, Overspill) are very valuable in this respect, but may have unwanted consequences or even make the resulting theories incompatible with initial metamathematical considerations. We assess both positive and negative implications, and extract contributions to Reverse Mathematics from a variety of examples, taken from nonstandard analysis and from logic and phase transitions.

[†]The second author will present the contributed talk. The authors wish to thank professor Ulrich Kohlenbach (Technische Universität Darmstadt) and professor Andreas Weiermann (University of Ghent) for their valuable advice.

▸ BERNHARD IRRGANG, *Applications of higher-dimensional forcing*.
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I will report on my project to develop a theory of higher-dimensional forcing iterations. Classical iterated forcing with finite support as introduced by Solovay and Tennenbaum works with continuous, commutative systems of complete embeddings which are indexed along a well-order. The idea of my approach is not to consider a linear system of embeddings but a higher-dimensional system indexed along a simplified morass. Results are:

(1) There exists a ccc forcing of size ω_1 which adds an ω_2-Suslin tree.

(2) There exists consistently a 0-dimensional Hausdorff space with spread ω_1 and size $exp(exp(\omega_1))$.

Moreover, it yields new proofs for:

(3) There exists consistently a chain $\langle X_\alpha \mid \alpha < \omega_2 \rangle$ such that $X_\alpha \subseteq \omega_1$, $X_\beta - X_\alpha$ is finite and $X_\alpha - X_\beta$ has size ω_1 for all $\beta < \alpha < \omega_2$ (P. Koszmider).

(4) $\omega_2 \not\to (\omega; 2)_\omega$ is consistent (S. Todorcevic).

These results were first proved with S. Todorcevic's method of ordinal walks.

[1] B. Irrgang, *Morasses and finite support iterations*, accepted for the ***Proceedings of the AMS***

[2] B. Irrgang, *Morasses and finite support iterations II*, submitted for the ***Annals of pure and applied logic***

▸ ALEKSANDER IWANOW, *Topological completions of ω-categorical groups*.
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A subgroup H of an infinite group G is called *inert* in G if for every $g \in G$, $H \cap gHg^{-1}$ is of finite index in H. We have found that any finite subgroup of an ω-categorical group is contained in an infinite residually finite inert subgroup.

Let G be countably categorical and H be an infinite residually finite inert subgroup of G. Let $X = \{gK : K$ is commensurable with H and $g \in G\}$. Then the action of G on X by left multiplication defines an embedding of G into $Sym(X)$ such that the closure of G in $Sym(X)$ is a locally compact group $\bar{G}$ and the closure of any $K < G$ commensurable with H is a compact subgroup of $\bar{G}$. We study how properties of $\bar{G}$ are connected with model theoretic properties of G. In particular we study measurable subsets of $\bar{G}$. Our main results describe the cases when $\bar{G}$ has uncountable cofinality and when $\bar{G}$ is ω-categorical.

▸ IGNACIO JANÉ, *Cantor's two views of ordinal numbers.*
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When in *Grundlagen* (1883) Cantor extended the sequence of the positive integers beyond the finite, he described the finite and the transfinite numbers as being obtained by means of two generating principles. The first principle provides the immediate successor of any generated number, while the second yields the limit of any definite totality of generated numbers with no largest element. From this characterization it immediately follows that no definite totality contains all numbers. This consequence was essential for Cantor's claim that the extended number sequence is absolutely infinite.

Although numbers were not introduced as ordinals, Cantor pointed out that each well-ordered set is isomorphic to the set of predecessors of a unique number under the generating order. He took this number (*Zahl*) as the ordinal (*Anzahl*) of that set. Soon after *Grundlagen* Cantor enhanced the ordinal role of numbers by redefining them as order-types of well-ordered sets. This new characterization, he said, is *purely mathematical*. From then on, Cantor disregarded the generating conception of number and kept to the ordinal definition.

Perhaps the ordinal way to numbers is purely mathematical. But discarding generation barred a natural path to block the set-theoretical paradoxes. Cantor explained them away as resting on a confusion between sets and inconsistent multiplicities. The elements of the latter, he said, do not coexist, but he failed to convince even Hilbert and Dedekind of the soundness of this distinction. One paradox involved the totality of all ordinals. Had Cantor kept to his original characterization of numbers, he could have explained what their lack of coexistence means and why their totality is not a set. In the *Grundlagen* setting, the ordinals do not all coexist because the two generating principles have no closure, i.e. because there is no definite totality closed under them both.

▸ ADIB BEN JEBARA, *About a distance for elementary particles.*
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In "An interpretation about space and time in quantum mechanics" in Logic Colloquium 2007, we have seen that the usual distance is not defined for space and time because we apply the theory ZFU (set theory with urelements). We explained some correlations between particles.

I would have doubts about the assumptions if I did not apply them for space successfully to explain the Big Crunch and the Big Bang in http://adibjebara.blogspot.com/ As time is but another dimension, I apply ZFU to time as well.

Now, let U1xU2 be space time with U1 space and U2 time.

Many things are discontinuous in quantum mechanics. So, why not space and time ? At the level of elementary particles, space and time are also not linearly ordered. As the experiment set up gives us meters and seconds, that means that the space and time considered by the experiment set up are the space and time at our level.

I conjecture that when we measure, we introduce, finitely, order of urelements in U1 and U2, in a laboratory. A distance is number of urelements in between + 1 which is converted to the measurement at our level in meters and seconds. The experiment set up changes much what exists as space and time.

The skeptics should be aware that some experiments needed interpretations and I provided them.

As a Platonist, I see some reality in Dedekind cardinals and, thus, in ZFU.

There is already an experiment where a particle goes both two separate ways. So, its space is different from ours.

There remain to find other experiments to detect that locations and times are not linearly ordered. There remains also to comment on the lack of localism.

I thank Mr Andereas Blass and Mr Abderazak Abadlia for their comments.

▸ REINHARD KAHLE, *Understanding functional self-application*.
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While functional self-application, i.e., the possibility of a function to be its own argument, is usually not considered in mathematics, it turned out that this concept can be meaningfully integrated in logical systems and programming languages. In mathematical logic it was first studied in the context of *combinatory logic* introduced by Schönfinkel in 1920 [7] and later on developed by Curry [3, 4], as well as by Church [1, 2] in the twin theory of λ *calculus*. It is an essential feature of type-free functional programming languages, but also of logical frameworks like Feferman's systems of *explicit mathematics* [5, 6]. Moreover, from the theoretical point of view, self-application can be used to prove standard *undefinability results*.

In this talk we show how functional self-application can be considered as a special case of *diagonalization*. In this perspective, it looses all its mystery; in particular, the *recursion-theoretic* reading of functional self-application allows to understand it as a well-known concept, used not only in logic, but implicitly also in every modern computer.

[1] Alonzo Church, *A set of postulates for the foundations of logic*, ***Annals of Mathematics (2)***, vol. 33 (1932), no. 2, pp. 346–366.

[2] ———, *A set of postulates for the foundations of logic (second paper)*, ***Annals of Mathematics (2)***, vol. 34 (1933), no. 4, pp. 839–864.

[3] Haskell Curry and Robert Feys, ***Combinatory Logic***, vol. I, North-Holland, 1958.

[4] Haskell Curry, J. Roger Hindley, and Jonathan Seldin, ***Combinatory Logic***, vol. II, North-Holland, 1972.

[5] Solomon Feferman, *A Language and Axioms for explicit Mathematics*, ***Algebra and Logic*** (J. Crossley, editor), Lecture Notes in Mathematics, vol. 450, Springer, 1975, pp. 87–139.

[6] ———, *Constructive Theories of Functions and Classes*, ***Logic Colloquium 78*** (M. Boffa, D. van Dalen, and K. McAloon, editors), North-Holland, 1979, pp. 159–224.

[7] Moses Schönfinkel, *Über die Bausteine der mathematischen Logik*, ***Mathematische Annalen***, vol. 92 (1924), pp. 305–316. German original of [8].

[8] ———, *On the Building Blocks of Mathematical Logic*, ***From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*** (Jean van Heijenoort, editor), Harvard University Press, 1967, pp. 355–366. English translation of [7].

▸ ANASTASIYA KARPENKO, *Weak interpolation property in $NE(K4)$*.
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A *normal modal logic* is any set of modal formulas containing all the classical tautologies and the axioms $\Box(A \to B) \to (\Box A \to \Box B)$ and closed under the rules of modus ponens, necessitation and substitution. The logic $K4$ is defined by adding one axiom scheme $\Box A \to \Box\Box A$. By $NE(L)$ we denote the set of all normal modal logics containing L.

If $\mathbf{p}$ is a list of propositional variables, let $A(\mathbf{p})$ denote a formula whose all variables are in $\mathbf{p}$. Let L be a logic, $\vdash_L$ a consequence relation associated with L. Suppose that $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are disjoint lists of propositional variables. We define the *weak interpolation property:*

WIP: If $A(\mathbf{p}, \mathbf{q}),\ B(\mathbf{p}, \mathbf{r}) \vdash_L \bot$, then there exist a formula $C(\mathbf{p})$ such that $A(\mathbf{p}, \mathbf{q}) \vdash_L C(\mathbf{p})$ and $C(\mathbf{p}),\ B(\mathbf{p}, r) \vdash_L \bot$.

Larisa Maksimova proved that WIP is equivalent to some weak variant of amalgamation. A class V has the *amalgamation property* if it satisfies the condition:

AP: For any $\mathcal{B}, \mathcal{C} \in V$ with a common subalgebra $\mathcal{A}$, there exist an algebra $\mathcal{D}$ in V and monomorphisms $\delta : \mathcal{B} \to \mathcal{D}$ and $\varepsilon : \mathcal{C} \to \mathcal{D}$ such that $\delta(x) = \varepsilon(x)$ for all $x \in \mathcal{A}$.

It is well known that there is a duality between normal modal logic and varieties of modal algebras. The K4 logic is defined by the variety of transitive modal algebras. For a given logic L, its associated variety $V(L)$ is determined by identities $A = \top$ for A provable in L.

For a class V of algebras, $FG(V)$ denotes the class of finitely generated algebras of V. An algebra is said to be *simple* if it has exactly two congruences.

Theorem 1 [1]: For any normal modal logic L the following are equivalent:

1) L has WIP;

2) the class $Sim(V(L))$ of simple algebras of $V(L)$ has AP;

3) the class $FG(Sim(V(L)))$ of finitely generated and simple algebras of $V(L)$ has AP.

By V_0 we denote a two-element transitive modal algebra with $\Box x = 1$.

For an arbitrary $n > 0$ let us define a topoboolean algebra V_n. The algebra V_n has n atoms $a_1, \ldots, a_n$, and $\Box x = \begin{cases} 1, & \text{if } x = 1; \\ 0, & \text{if } x \neq 1. \end{cases}$

Proposition: $FG(Sim(V(K4))) = \{V_n | n \geq 0\}$.

Theorem 2: Let L be a logic over $K4$. Then L has WIP iff one of the following conditions is valid:

1) $V_n \in V(L)$ for all $n > 0$;

2) $V_3 \notin V(L)$.

Theorem 3: WIP is decidable over $K4$.

[1] Maksimova L., *On a form of interpolation in modal logic*, ***Logic Colloquium 2005, Bulletin of Symbol Logic***, vol. 12, no. 2 (2006), p. 340.

▸ NEIL KENNEDY, *Modal modal logic.*
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In this paper, I will present a multi-modal framework that allows one to express a notion of modality that not only applies to worlds but also to accessibility relations. To illustrate the idea, suppose we have a language in which there are alethic and epistemic modalities $\Box$ and K respectively, and we want to express something like possible knowledge. One could think that the correct formalisation of this notion is given by the expression $\Diamond K$. However, upon closer analysis, this expression only picks up the following reading: the possibility that the same agent knows in a different world. Another reading, which seems just as important, is not expressible in this language: the possibility that a different agent knows in the same world. To express this latter reading, one must develop a language in which the scope of the modality can be made clear (i.e. if the modality applies to the agent or the worldly matters), and, most importantly, one must also specify what kind of semantics a language of this sort requires.

If we limit ourselves to the modalities above, we can disambiguate between the two readings by use of parentheses: $\Diamond K$ for the first reading, and $(\Diamond)K$ for the second. In order to capture all the appropriate properties of modalities in the second reading, we must also allow any string of negations and modalities (e.g. $\neg\neg\Diamond\neg\Diamond$) to occur in parentheses. Semantics for these expressions will be given with models of the form $< W, U, V, (R_u)_{u\in U}, (S_v)_{v\in V}, \rho, \sigma, val >$ where: (1) W is the base world, (2) val is a valuation on W, (3) $(R_u)_{u\in U}$ is the set of possible accessibility relations of the first modality (indexed by U), (4) similar definition for $(S_v)_{v\in V}$, (5)ρ and σ are binary relations on V and U respectively, they represent the accessibility relation between sets of possible accessibility relations of the first and second modalities respectively.

A labelled tableau calculus for this language will also be given.

▸ VALERY KHAKHANIAN, *The independence of strong uniformization from Church Thesis with choice in the set theory with extensionality and intuitionistic logic.*
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Let CT and CT! be Church Thesis with choice (strong) and with uniqueness (weak) respectively. Let U and U! be principles strong and weak uniformization. Let ZFI2C + DCS (all postulates here or about are in [1] and [2])be the set theory with intuitionistic logic and two kinds of variables (natural and set) and with collection scheme. It is known that: a)ZFI2C + CT! $\vdash$ U!; b)ZFI2C + DCS + CT! $\nvdash$ CT; c) ZFI2C - extensionality + DCS + CT $\nvdash$ U. We proved that d)ZFI2C + DCS + CT $\nvdash$ U. Corollaries: e) ZFI2C + DCS + U! $\nvdash$ U; f) ZFI2C + DCS + CT! $\nvdash$ U.

I am going to speak about some properties of models of realizability types which were used to prove d).

[1] Hahanjan V.H. The comparative strength of variants of Church Thesis at the level of set theory/ Soviet Math. Dokl., vol.21, 1980, no.3, P. 894-898.

[2] Hahanjan V.H. The consistency of intuitionistic set theory with formal mathematical analysis/ Soviet Math. Dokl., vol. 23, 1980, no.1, P. 46-50.

▸ LESZEK ALEKSANDER KOŁODZIEJCZYK, *The Σ_1 collection principle over finite fragments of bounded arithmetic.*
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Let T^n_2 be the usual Buss-style theory axiomatized by the induction scheme for Σ^b_n formulae. We show that if T^n_2 does not prove the weak pigeonhole principle for PV_n functions (functions from the n-th level of the polynomial hierarchy), then the Σ_1 collection principle $B\Sigma_1$ is not finitely axiomatizable over T^n_2. The proof is based on the construction of a model of $T^n_2 + \neg WPHP(PV_n)$ in which a given finite fragment of $B\Sigma_1$ holds, but full $B\Sigma_1$ would imply that the polynomial hierarchy "almost collapses" in a way which is incompatible with the failure of $WPHP$.

Completely analogous results hold with the theory S^n_2 in place of T^n_2.

▸ FRANZ-VIKTOR KUHLMANN, *The model theory of tame valued fields.*
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I will summarize my results about completeness, model completeness and decidability of tame fields relative to their value groups and residue fields, and a corresponding result about separably tame fields. Tame (or separably tame, resp.) fields are henselian valued fields K for which the ramification field of the extension $K^{sep}|K$ is algebraically closed (or separable-algebraically closed, resp.); here, K^{sep} denotes the separable-algebraic closure of K. The class of tame fields includes the class of Kaplansky fields, for which relative completeness, model completeness and decidability have been shown by Ershov and, independently, Ziegler. In contrast to these fields and all other classes of valued fields for which such results had been known, the maximal immediate extensions of tame fields are in general not unique. To balance this loss of an important tool, I had to prove much more advanced theorems about valued fields with residue fields of positive characteristic.

All tame fields are perfect, and all algebraically maximal perfect fields of positive characteristic are tame. Since algebraic maximality is necessary to obtain good model theoretic results, the model theory of tame fields represents the model theory of perfect valued fields in positive characteristic.

The model theory of tame fields has nice applications to the theory of places of algebraic function fields in arbitrary characteristic.

▸ ROMAN KUZNETS, *Complexity through tableaux in justification logic*.
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Logics J, JD, JT, J4, JD4, and LP are explicit counterparts of modal epistemic logics K, D, T, K4, D4, and S4 respectively (see [2, 1] for more details). An upper bound of Σ_2^p for the satisfiability problem in these justification logics was announced in [3]. The algorithm was essentially a propositional tableau procedure (though not presented as such) followed by a check on possible contradictions among justifications. The former part is traditionally NP while the latter was shown to be co-NP, thus yielding Σ_2^p as the total complexity.

Recently an omission was found in the proof for two of the six logics: JD and JD4. Restoring the upper bound for JD called for the use of prefixed tableaux. Below we provide the tableau-style formulation of the first part of the original algorithm for J, JT, J4, and LP as well as a new tableau algorithm for JD that help establish the Σ_2^p upper bound for satisfiability problems in all these 5 logics (note the integer prefixes used for JD).

J, J4	$\dfrac{T\ s{:}G}{T\ {*}(s,G)}$	$\dfrac{F\ s{:}G}{F\ {*}(s,G)}$
JT, LP	$\dfrac{T\ s{:}G}{\begin{array}{l}T\ G\\ T\ {*}(s,G)\end{array}}$	$\dfrac{F\ s{:}G}{F\ {*}(s,G)\ \mid\ F\ G}$
JD	$\dfrac{n\ T\ s{:}G}{\begin{array}{r}n\ T\ {*}(s,G)\\ n{+}1\ T\ G\end{array}}$	$\dfrac{n\ F\ s{:}G}{n\ F\ {*}(s,G)}$

Whether the same bound holds for JD4 remains at this point an open problem.

[1] Sergei Artemov, *Justification logic*, Technical Report TR-2007019, CUNY Ph.D. Program in Computer Science, 2007.

[2] Vladimir N. Brezhnev, *On explicit counterparts of modal logics*, Technical Report CFIS 2000–05, Cornell University, 2000.

[3] ROMAN KUZNETS, *On the complexity of explicit modal logics*, ***Proceedings of CSL 2000*** (Fischbachau, Germany), (Peter Clote and Helmut Schwichtenberg, editors), ***Lecture Notes in Computer Science***, vol. 1862, Springer, 2000, pp. 371–383.

▸ GRAHAM E. LEIGH AND MICHAEL RATHJEN, *An ordinal analysis of theories of truth.*

We augment the first-order language of Peano Arithmetic with a unary predicate T with $T(x)$ intended to mean "x is the Gödel number of a true sentence of arithmetic". In [2] Friedman and Sheard constructed a list of twelve axioms and rules of inference concerning the predicate T, each expressing some desirable property of truth, and classified all subsets of the list as either consistent or inconsistent. This gave rise to a collection of nine theories of truth, two of which have been treated in the literature (see [2], Halbach [3], Sheard [4] and Cantini [1] for more details). We uncover the proof-theoretic strength of the remaining seven and in the process construct a proof-theory of truth allowing the systems to be subject to an ordinal analysis.

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[2] H. Friedman and M. Sheard, An axiomatic approach to self-referential truth, Annals of Pure and Applied Logic, 33 1–21, 1987.

[3] V. Halbach, A system of complete and consistent truth, Notre Dame Journel of Formal Logic, vol. 35, 3, 311-327, 1994.

[4] M. Sheard, Weak and strong theories of truth, Studia Logica, 68, 89–101, 2001.

▸ DANIEL LEIVANT, *Machine models for the arithmetical and analytical hierarchies.*
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We show that natural generalizations of Turing machines can capture the arithmetical hierarchy, the hyper-arithmetical sets, and the analytical hierarchy. All one needs is to consider a broader semantic interpretation of Chandra-Kozen-Stockmeyer's 1980 alternating Turing machines (where non-determinism can be used both existentially and universally).

Such machines are traditionally defined in terms of local branching conditions, and consequently are no more powerful than deterministic machines. We generalize the definition of acceptance by alternating machines by referring to more global conditions on existential and universal branching. We show that such machines accept precisely the inductive (i.e. Π^1_1) sets. Alternating deciders, which either accept or reject each input, accept the hyper-elementary (Δ^1_1) sets. When a uniform bound k is imposed on alternations along every execution path, we obtain level k of the arithmetical hierarchy, in perfect analogy with the characterization of the polynomial-time hierarchy in terms of time-bounded alternating Turing machines. Finally, further equipping machines with negation states (endowed with a strong semantics), yields precisely the Π^1_{k+1} sets when such states occur up to k times along every execution path.

We further show how these characterizations shed light on classical results of Higher Recursion Theory.

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The notion of an abstract elementary class is obtained from that of the elementary classes of classical model theory by abandoning syntax and extracting their purely category-theoretic content: an AEC is a class of structures equipped with a strong

submodel relation whose behavior mimics that of the elementary submodel relation (unions of chains, coherence, a downward Lowenheim-Skolem property, and so on). In this more general context, we speak not of the syntactic type of an element a over a set A, but rather of its Galois type over a model M in the AEC—roughly speaking, the orbit of a under automorphisms of the monster model that fix M.
We present a method of topologizing sets of Galois types over structures in AECs with amalgamation. Although the topological spaces thus produced are not, strictly speaking, generalizations of the spaces of syntactic types familiar from first order logic, they provide similar benefits. In particular, there are natural correspondences between topological properties of the spaces and semantic properties of the AEC (tameness of Galois types, for example, emerges as a separation principle). Moreover, the topologies thus defined give rise to a notion of Cantor-Bendixson rank and, most interestingly, to a family of Morley-like ranks which show some promise as tools for analyzing the stability spectra of AECs. We sketch a few results along these lines.

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Epistemic logics are usually approached as a special kind of modal logics - possible worlds represent epistemic states (cf. [2]), accessibility relation gives epistemic alternatives and properties of necessity operator (K, T, S4, S5) correspond in this setup to properties of agent's epistemic states. There have been numerous discussions about adequacy or inadequateness of resulting systems of epistemic logic, the main objection was, that these systems are too far away from representing epistemic states of real agents.

The aim of our paper is to discuss relevant logic (see e.g. [1]) as a framework which allows more 'realistic' alternatives. We use relational semantics for relevant logic introduced by Routley-Meyer [4] and developed by Restall [3] and others. This semantics is a generalization of standard Kripke semantics; it allows incomplete and inconsistent states and employs two accessibility relation a ternary R and binary C responsible for (relevant) implication and negation respectively. We explore formal properties of relevant epistemic framework and compare it to those obtained in the standard modal approach.

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[4] Routley, R., R.K. Meyer, V. Plumwood and R. Brady, ***Relevant Logics and its Rivals***, Volume I, Atascardero, CA: Ridgeview, 1983.

▸ LARISA MAKSIMOVA, *Interpolation and related properties in semilattice based varieties.*
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Let us fix an arbitrary signature consisting of functional symbols and constants and including $\wedge$. We consider varieties of algebras, where $\wedge$ is a greatest lower bound; as usual, $x \leq y \iff x \wedge y = x$. We define a variant of interpolation property and find its algebraic equivalent.

There is a close connection between different versions of interpolation and Beth properties in varieties of algebras and amalgamation and epimorphisms surjectivity. One can find the definitions of amalgamation property AP, super-amalgamation SupAP, strong amalgamation StrAP, restricted amalgamation RAP, strong epimorphisms surjectivity SES and of their algebraic equivalents, namely, Robinson property ROB* (originated from H.Ono 1986), restricted interpolation IPR, projective Beth property PBP in [1,2]. We recall the definition of Super-amalgamation property:

(SupAP) For any $\mathbf{A}, \mathbf{B}, \mathbf{C} \in V$ such that $\mathbf{A}$ is a common subalgebra of the algebras $\mathbf{B}$ and $\mathbf{C}$, there exist an algebra $\mathbf{D}$ in V and monomorphisms $g : \mathbf{B} \to \mathbf{D}$ and $h : \mathbf{C} \to \mathbf{D}$ such that $g(x) = h(x)$ for all $x \in \mathbf{A}$ and, moreover,

$$g(x) \leq h(y) \iff (\exists z \in \mathbf{A})(x \leq z \text{ and } z \leq y),$$

$$g(x) \geq h(y) \iff (\exists z \in \mathbf{A})(x \geq z \text{ and } z \geq y).$$

(WSupAP) arises from (SupAP) by changing "monomorphisms" to "homomorphisms".

Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ be pairwise disjoint lists of variables. We formulate *Inequalities interpolation principle*:

(IIP) Let $\Gamma(\mathbf{x}, \mathbf{y})$ and $\Delta(\mathbf{x}, \mathbf{z})$ satisfy the condition $\Gamma'(\mathbf{x}) = \Delta'(\mathbf{x})$, where $\Gamma'(\mathbf{x}) = \{\alpha(\mathbf{x}) | \, \Gamma(\mathbf{x}, \mathbf{y}) \models_{\mathbf{V}} \alpha(\mathbf{x})\}$ and $\Delta'(\mathbf{x}) = \{\alpha(\mathbf{x}) | \, \Delta(\mathbf{x}, \mathbf{z}) \models_{\mathbf{V}} \alpha(\mathbf{x})\}$. If $\Gamma(\mathbf{x}, \mathbf{y}), \Delta(\mathbf{x}, \mathbf{z}) \models_V u(\mathbf{x}, \mathbf{y}) \leq v(\mathbf{x}, \mathbf{z})$, then there is a term $t(\mathbf{x})$ such that $\Gamma(\mathbf{x}, \mathbf{y}) \models_V u(\mathbf{x}, \mathbf{y}) \leq t(\mathbf{x})$ and $\Delta(\mathbf{x}, \mathbf{z}) \models_V t(\mathbf{x}) \leq v(\mathbf{x}, \mathbf{z})$.

Theorem 1. *For any variety V:*
(i) IIP is equivalent to WSupAP; (ii) V has SupAP iff it has AP and WSupAP.

Proposition 2. (i) $IIP \Rightarrow PBP$; (ii) $WSupAP \Rightarrow (SES \ \& \ RAP)$.

Proposition 3. *Let a variety V satisfy the condition: there are a term $\varepsilon(x, y)$ and a constant $\mathbf{e}$ such that $x = y \models_V \mathbf{e} \leq \varepsilon(x, y)$ and $\mathbf{e} \leq \varepsilon(x, y) \models_V x = y$. Then IIP $\Rightarrow$ AP, and so IIP, SupAP and WSupAP are equivalent.*

[1] Gabbay D. M., Maksimova L., ***Interpolation and definability: modal and intuitionistic logics***, Oxford Univ. Press, 2005.

[2] Maksimova L., *Restricted interpolation and projective Beth property in equational logic*, ***Algebra and Logic***, vol. 42 (2003), pp. 712–726.

► MACIEJ MALICKI, *Isometry groups of separable metric spaces.*
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The talk will be concerned with representing groups as full isometry groups of metric spaces so that nice properties of the group are reflected by nice properties of the metric space. It is easy to see that if X is a Polish metric space, that is, a complete metric space whose metric induces a separable topology, then the isometry group of X, $\mathrm{Iso}(X)$, considered with the pointwise convergence topology, is Polish. Again, it is an easy observation that $\mathrm{Iso}(X)$ is compact provided that X is compact. It was proved in [1] that $\mathrm{Iso}(X)$ is locally compact if X is proper, that is, if all closed balls of (X, d) are compact. A natural question arises whether the converses to these facts hold.

In [1], Gao and Kechris showed that every Polish group is indeed isomorphic to the isometry group of some Polish space. Then Melleray [3] found a simpler proof of their result and used it to prove that every compact group is isomorphic to the isometry group of a compact space. In [2], we provide the last missing piece of the picture by showing that every locally compact Polish group is the isometry group of a proper Polish space. This solves a problem posed by Gao and Kechris in [1, p.76].

If time permits, I will comment on the tools used in the proof of this result, and say a couple of words about ultrametric spaces and their isometry groups.

This is a joint work with Slawek Solecki.

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▸ JOSÉ M. MÉNDEZ AND GEMMA ROBLES AND FRANCISCO SALTO, *Semantics for relevant logics plus the disjunctive syllogism*.
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As is known, the disjunctive syllogism (d.s) (i.e., the rule if $\vdash A$ and $\vdash \neg A \vee B$, then $\vdash B$) is not a rule of relevant logics. It is, however, an admissible rule in them. The simplest proof of its admissibility is maybe the "normalization" procedure due to Routley and Meyer (cf. [1], [3]). The idea is to define "normal models" to prove that d.s holds in normal models, and finally, to show that any theorem of the relevant logic under consideration is valid in any model iff it is valid in normal models.

The aim of this paper is to provide a semantics for d.s and not only a proof of its admissibility, in particular, to provide a (Routley-Meyer) semantics for any logic including Contractionless Ticket Entailment TW (cf. [2]) plus d.s but without t and $\circ$.

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[3] R. Routley et al., *The semantics of Entailment IV* , in R. Routley et al., ***Relevant Logics and their Rivals***, 1, pp. 407-424, Atascadero, CA: Ridgeview Publishing Co., 1982.

▸ GRIGORI MINTS, *Cut Free Formulation of a Quantified Logic of Programs*.
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A predicate extension $\mathbf{SQHT}^{=}$ of the Smetanich's logic of here-and-there had been introduced by V. Lifschitz, D. Pearce, and A. Valverde (A characterization of strong equivalence for logic programs with variables, Procedings of International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR), 2007) to characterize the notion of strong equivalence of logic programs with variables and equality with respect to stable models. The semantics for this logic is determined by intuitionistic Kripke models $M =< W, R, D >$ where $W = \{h, t\}$ R is reflexive and hRt, and D is a non-empty constant individual domain. Equality is decidable. Derivable objects of our formulation are *hyperequents* $\Gamma \Rightarrow \Delta * \Pi \Rightarrow \Phi$ to be read: $\Gamma \Rightarrow \Delta$ here or $\Pi \to \Phi$ there.

Translation into formulas: $\&\Gamma \to \vee\Delta \vee (\&\Pi \to \neg\neg \vee \Phi)$.

The system has familiar axioms, classical Gentzen rules for all connectives in $\Pi \Rightarrow \Phi$ and for all connectives except $\Rightarrow\to, \Rightarrow \neg$ in $\Gamma \Rightarrow \Delta$. Special rules for $\Rightarrow_h\to, \Rightarrow_h \neg$, equality (in addition to familiar rules) and h-t-transfer:

$$\frac{A, \Gamma \Rightarrow \Delta, B * \Pi \Rightarrow \Phi \qquad \neg B, \Gamma \Rightarrow \Delta, \neg A * \Pi \Rightarrow \Phi}{\Gamma \Rightarrow \Delta, A \to B * \Pi \Rightarrow \Phi}$$

$$\frac{\Gamma \Rightarrow \Delta, *A, \Pi \Rightarrow \Phi}{\Gamma \Rightarrow \Delta, \neg A * \Pi \Rightarrow \Phi} \quad \frac{t = s, \Gamma \Rightarrow \Delta * t = s, \Pi \Rightarrow \Phi}{\Gamma \Rightarrow \Delta * t = s, \Pi \Rightarrow \Phi} \quad \frac{A, \Gamma \Rightarrow \Delta * A, \Pi \Rightarrow \Phi}{A, \Gamma \Rightarrow \Delta * \Pi \Rightarrow \Phi}$$

Theorem 1. *The rules are sound and complete.*

Existence is definable: $\exists x Px \iff \forall y(\forall x(Px \to Py) \to Py)$.

Let **WEM** be an extension of the intuitionistic logic with $\neg p \vee \neg\neg p$,decidable equality, the formula of constant domains and the equivalences for moving $\neg$ through $\forall, \exists$.

Theorem 2. **SQHT** $=$ *is conservative over* **WEM** *for the formulas without embedded positive occurrences of implication.*

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My aim is to examine the weaker systems of relevant logic. There are at least two approaches to the problem. First of all, it is possible to restrict a validity of some theorems. Thus, in the logic S of Martin and Meyer we have just two axioms, it means:

$(\beta \to \gamma) \to [(\alpha \to \beta) \to (\alpha \to \gamma)]$ (suffixing),

$(\alpha \to \beta) \to [(\beta \to \gamma) \to (\alpha \to \gamma)]$ (transitivity),

and the rule of modus ponens. The next example is Brady's logic DJ^d in which we have a weaker form of transitivity:

$[(\alpha \to \beta) \wedge (\beta \to \gamma)] \to (\alpha \to \gamma)$,

and the *law of contraction*:

$[(\alpha \to (\alpha \to \beta)] \to (\alpha \to \beta)$

is not valid.

However, this is not the only possible approach. We can investigate even weaker logics which have no theorems and are characterised only by rules of deducibility. In this way D.M. Gabbay introduced systems $\vdash_0$ and $\vdash_1$ and Font with Rodriguez the deductive system WR. The well known system R is an axiomatic extension of WR. We can say that WR is the deductive system determined by the following conditions:

(1) $\vdash_{WR}$ is finitary,

(2) WR has no theorems,

(3) For every $\alpha, \alpha_1, ..., \alpha_n \in Fm$, $\{\alpha_1, ..., \alpha_n\} \vdash_{WR} \alpha$ iff $\vdash_R (\alpha_1 \wedge, ..., \wedge \alpha_n) \to \alpha$.

Following this idea it is possible to introduce other deductive systems like WRM, WE etc. Such systems are not protoalgebric and since every algebraizable deductive system is also protoalgebric, it follows that they are not algebraizable either.

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Strong provability *F is true and provable in Peano Arithmetic* is known ([3]) to be propositionally axiomatized by Grzegorczyk's logic Grz, which postulates are classical logic axioms and rules, $\Box(F \to G) \to (\Box F \to \Box G)$, $\Box F \to \Box\Box F$, $\Box F \to F$, $\Box(\Box(F \to$

$\Box F) \to F) \to F$, and Rule of Necessitation: $\vdash F \Rightarrow \vdash \Box F$. Artemov's Logic of Proofs LP ([2]) axiomatizes proof operators *t is a proof of F in Peano Arithmetic*. The postulates of LP are the ones of the classical propositional logic, $t{:}(F \to G) \to (s{:}F \to (t\cdot s){:}G)$, $t{:}F \to F$, $t{:}F \to !t{:}(t{:}F)$, $s{:}F \to (s+t){:}F$ and $t{:}F \to (s+t){:}F$, and the rule of inferring $c{:}A$ whenever A is an axiom and c is a proof constant.

Joint logics of proofs and provability have been studied in a variety of languages ([1, 4, 5, 6, 7]). In this paper we found the arithmetically complete logic of strong provability and explcit proofs GrzA in the joint language of Grz and LP. GrzA consists of axioms and rules of Grz and LP together with axioms $t{:}P \to \Box P$ and $\neg t{:}P \to \Box \neg t{:}P$. We show that GrzA is complete with respect to the arithmetical provability interpretation, supplied with a corresponding Fitting epistemic semantics and enjoys some sort of a finite model property, which also yields the decidability of GrzA for any given finite specification of constants.

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[2] ——— *Explicit provability and constructive semantics*, ***Bulletin of Symbolic Logic***, vol. 7 (2001), no. 1, pp. 1–36.

[3] G. Boolos, ***The Logic of Provability***, Cambridge University Press, 1993

[4] E. Nogina, ***Logic of proofs with the strong provability operator***, *ILLC Prepublication Series ML-94-10*, Institute for Logic, Language and Computation, University of Amsterdam, 1994.

[5] ——— *On logic of proofs and provability*, ***Bulletin of Symbolic Logic***, vol. 12 (2006), no. 2, pp. 356.

[6] ——— *Epistemic completeness of* GLA, ***Bulletin of Symbolic Logic***, vol. 13 (2007), no. 3, pp. 407.

[7] T. Yavorskaya (Sidon), *Logic of proofs and provability*, ***Annals of Pure and Applied Logic***, vol. 113 (2001), no. 1–3, pp. 345-372.

► CYRUS F NOURANI, *Positive Realizability on Horn Filters.*
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During 2005-2007 the author explored positive and Horn fragment categories based on his 1981 on positive forcing on Kiesler fragments. Positive forcing had defined T* on a theory T to be T augmented with induction schemas on the generic diagram functions. for a model M for T. Let P be a poset, F a family of sets, G subset of P. Suppose positive forcing is defined on P. The author defined positive local realizability on 2005-2007 publications (ASL).

Theorem 1. *(1981) The positive forcing T* is a F-generic filter.*

Lemma T* is a principal proper filter.

Proposition Let I be the set T*. Let $\phi(x1 \ldots xn)$ be a Horn formula and let $\Re i \quad i \in I$ be models for language L. Let $a1 \ldots an \in \Pi \in IAi$. The $\Re i$ are fragment Horn models. If $\{i \in I : \Re i \models \phi[(a1(i) \ldots an(i)]\}$ then the direct product $\Pi_D \Re I \models \phi[(a1(i)D \ldots an(i)D]$, where D is the generic filter on T*.

Completability theorem at T* on universal Horn sentences might be provable without CH: $2^\omega = \omega^+$.

Applying positive local realizability from 2,3:

Theorem 2. *Let $(P, \leq)$ be a positive Horn poset and $p \in P$. If D is a countable family of dense subsets of P then $T * (P)$ is D-generic filter F in P such that $p \in F$ and every $p \in F$ has a positive local realization.*

The above theorem is a Horn density counterpart to the Rasiowa-Sikorski lemma.

1.Nourani,C.F., Functional generic filter, ASL, Montreal, May 2006.

2.Nourani,C.F., Functorial models and positive Realizability, ASL Florida, March 2007.

3. Nourani,C.F.,.Positive Omitting Types and Fragment Consistency,.Godel Society conference April 24-25 Brasil.

▸ ANVAR NURAKUNOV, MICHAł STRONKOWSKI, *A new proof Pigozzi theorem.*

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In universal algebra Baker theorem states that if a variety (equationally definable class) $\mathcal{V}$ in finite language is generated by finite number of finite algebras and all algebras in $\mathcal{V}$ have distributive lattices of congruences then $\mathcal{V}$ is strictly elementary [1]. Pigozzi generalized the above theorem to quasivarieties (quasi-equationally definable classes) [3], which was a significant improvement. Probably the easiest proof of Baker theorem was given by Baker and Wang [2]. It is based on the notion of definable principal subcongruence. We introduce the notion of definable relative principal subcongruence what allows us to carry over the proof to the quasivariety case. As the result we obtain an easy (probably the easiest) proof of Pigozzi theorem.

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▸ ISABEL OITAVEM, *Recursion schemes with pointers.*

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In this talk we discuss recursion-theoretic characterizations of classes of complexity. We explore the notion of pointers in the recursion schemes in order to reach non-determinism and parallelism.

▸ EUGENIO OMODEO, ALBERTO POLICRITI, *The Decidability of the Bernays Schönfinkel Ramsey Class for Set Theory.*

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As is well-known, the Bernays-Schönfinkel-Ramsey class of all prenex $\exists^*\forall^*$-sentences which are provable in first-order predicate calculus is decidable. This paper shows that an analogous result holds when the only available predicate symbols are $\in$ and $=$, no constants or function symbols are available, and one moves inside a (rather generic) set theory whose axioms yield the well-foundedness of membership and the existence of infinite sets.

▶ LUIZ CARLOS PEREIRA, EDWARD HERMANN HAEUSLER, VASTON GONCALVES DA COSTE, WAGNER SANZ, *Revisiting Peirces rule in Natural Deduction.*
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The introduction and elimination rules for material implication in natural deduction are not complete with respect to the implicational fragment of classical logic. A natural way to complete the system is through the addition of a new natural deduction rule corresponding to Peirces formula $(((A \to B) \to A) \to A)$:

$$\cfrac{\begin{matrix}[(A \to B)] \\ \vdots \\ A\end{matrix}}{A}$$

E. Zimmermann [3] has shown how to extend Prawitz normalization strategy to Peirces rule: applications of Peirces rule can be restricted to atomic conclusions. The aim of the present paper is to extend Seldins normalization strategy to Peirces rule by showing that every derivation Π in the implicational fragment can be transformed into a derivation Π' such that no application of Peirces rule in Π' occurs above applications of $\to$-introduction and $\to$-elimination. As a corollary of Seldins normalization strategy we obtain a weak form of Glivenkos theorem for the fragment $\to$.

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▶ DAVID PIERCE, *Induction and recursion.*
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My main purpose is to point out, and to ask why it is, that certain foundational matters are commonly misrepresented or misunderstood.

Dedekind [1, II.130] makes an observation overlooked by Peano [7] and others: A set with an initial element and a successor-operation may admit proof by induction without admitting inductive or rather *recursive* definition of functions.

Landau [3, Preface for the Teacher] confesses to having confused induction with recursion. Henkin [2] works out the distinction. Yet the confusion continues to be made, even in textbooks intended for students of mathematics and computer science who ought to be able to understand the distinction. Textbooks also perpetuate related confusions, such as suggestions that induction and 'strong' induction (or else the 'well-ordering principle') are logically equivalent, and that either one is sufficient to axiomatize the natural numbers.

In an exercise in one noteworthy textbook [5, II.1, p. 38], the reader is invited to show the logical independence of the three axioms introduced by Dedekind, but commonly called by the name of Peano: (α) the initial element is not a successor, (β) the successor-operation is injective, and (γ) proof by induction works. But first, just after the introduction of these as axioms for the natural numbers, these numbers are used to index iterates of functions. This indexing is used later (II.2) to define addition and multiplication. But this indexing strictly requires all three of the axioms, normally in the equivalent form introduced only later still (II.11) and called the Peano–Lawvere Axiom. (Mention of this is absent from later editions, as [6]; it is called the Dedekind–Peano Axiom in [4, 9.1, p. 156].)

Landau implicitly (and Henkin explicitly) shows that addition and multiplication can be defined by induction alone. But the argument takes some work. (Strictly, the argument requires that these operations are being defined on a *set;* but one can avoid this assumption.) If one thinks that the recursive definitions of addition and multiplication—$n+0=n$, $n+(k+1)=(n+k)+1$, $n\cdot 0=0$, $n\cdot(k+1)=n\cdot k+n$—are *obviously* justified by induction alone, then one may think the same for exponentation, with $n^0 = 1$, $n^{k+1} = n^k \cdot n$. However, while addition and multiplication are well-defined on $\mathbb{Z}/(n)$ (which admits induction), exponentiation is not; rather, we have $(x, y) \mapsto x^y \colon \mathbb{Z}/(n) \times \mathbb{Z}/\phi(n) \to \mathbb{Z}/(n)$. This is one example to suggest that getting things straight may make a pedagogical difference.

[1] Richard Dedekind, ***Essays on the theory of numbers. I: Continuity and irrational numbers. II: The nature and meaning of numbers***, Dover, 1963

[2] Leon Henkin, *On mathematical induction*, ***The American Mathematical Monthly***, vol. 67 (1960), no. 4, pp. 323–338

[3] Edmund Landau, ***Foundations of Analysis***, Chelsea, 1966

[4] F. William Lawvere and Robert Rosebrugh, ***Sets for mathematics***, Cambridge, 2003

[5] Saunders Mac Lane and Garrett Birkhoff, ***Algebra***, Macmillan, 1967

[6] ———, ***Algebra***, third ed., Chelsea, 1988

[7] Guiseppe Peano, *The principles of arithmetic, presented by a new method*, ***From Frege to Gödel***, (Jean van Heijenoort, editor), Harvard, 1967

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The Logics of Formal Inconsistency (LIFs) were proposed in [1] as a general family of paraconsistent logics which can internalize the notions of consistency and inconsistency at the level of object-language. Though a large variety of such logics having different syntactic features is developed in [1], it remains at the propositional level. Our present purpose is to extend these logics to a first-order predicate calculi. Besides their first-order axiomatization, we'll show an interpretation of the calculi in terms of quasi-structures (or pre-structures) which permit a paraconsistent approach to their semantics. Moreover, this approach makes possible to exhibit a Henkin-type [2] construction according to which the completeness of a wide family of first-order LIFs can be established.

[1] W. A. Carnielli, M. E. Coniglio, and J. Marcos. *Logics of formal inconsistency*, In D. Gabbay and F. Guenthner, editors, ***Handbook of Philosophical Logic***, v.14, pages 15-107. Kluwer Academic Publishers, 2nd edition, 2007.

[2] L. Henkin. *The completeness of the first-order functional calculus.* ***The Journal of Symbolic Logic***, v.14, n.3., pages 159-166, 1949.

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According to Ackermann (cf. [1]), "Das Absurde" is the meaning of the propositional falsity constant he introduces in his system Π' in order to define the modalities in it. Ackermann remarks that he introduces the modalities similarly as Johansson introduced negation in the "Minimalkalkül" (cf. [2]). Ackermann is, arguably, the first author presenting a logical system in which the falsity constant is not equivalent to *some* class of contradictions. Be it as it might be, we shall generally use Ackermann's term "Das Absurde" to refer to the meaning of the falsity constant.

The aim of this paper is to investigate some of the different faces of "Das Absurde". In particular, we shall define the minimal logics (and their extensions) in which "Das Absurde" is equivalent to:

1. Any contradiction
2. The negation of a theorem
3. The argument of a negation theorem
4. The negation of any self-identity
5. An unqualified falsehood.

[1] W. Acermann, *Begründung einer strenger Implication* , ***Journal of Symbolic Logic***, 21, pp. 113-128, 1956.

[2] I. Johansson, *Der Minimal Kalkül, ein reduzierte intuitionistischer Formalismus* , ***Compositio Mathematica***, pp. 119-136, 1936.

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Given a σ-ideal I of Borel sets in a Polish space we usually associate with I a forcing notion $\mathbb{P}_I = \mathrm{Bor}/I$. On the other hand, we say that a Borel function $f : X \to Y$ is σ-continuous if countably many (arbitrary) sets X_n can be found such that they cover X and $f{\restriction}X_n$ is continuous for each n. A particularly simple example of a Borel function which is not σ-continuous is the function P found by Pawlikowski. A σ-ideal of sets on which P is σ-continuous gives rise to a forcing notion (called Steprāns forcing). I will talk about this forcing and establish some its basic properties (e.g. continuous reading of names, Axiom A, etc.).

▸ MEHRNOOSH SADRZADEH AND ROY DYCKHOFF, *Cut-free sequent calculi for logics with adjoint modalities.*
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We consider modal logics where the underlying logic is non-classical and modalities come in adjoint pairs, but are not (in general) closure operators. Despite absence of negation and implication, and of axioms corresponding to the characteristic axioms of (e.g.) **T**, **S4** and **S5**, such logics are useful, as shown in [1], for encoding and reasoning about information and misinformation in interactive multi-agent systems.

The semantics of such logics can be determined by algebras (e.g. lattices or quantales) with agent-indexed families of adjoint pairs of operators.

We present here cut-free sequent calculi for such algebras (and thus for the logics). The calculi exploit structural operators in the style of Belnap [2], as illustrated in [3] for classical modal logics and in [4] for residuated monoids. Cut-admissibility, and thus the completeness of the calculi, is shown by constructive syntactic methods, as in [5].

[1] A. Baltag, B. Coecke, M. Sadrzadeh, *Epistemic Actions as Resources*, ***Journal of Logic and Computation***, vol. 17 (2007), no. 3, pp. 555–585.

[2] N. D. Belnap, Jr, *Display logic*, ***Journal of Philosophical Logic***, vol. 11 (1982), pp. 375–417.

[3] K. Brünnler, *Deep Sequent Systems for Modal Logic*, ***Advances in Modal Logic*** (Noosa, Queensland, Australia), (G. Governatori, I. M. Hodkinson and Y. Venema, editors), vol. 6, College Publications, 2006, pp. 107–119.

[4] M. Moortgat, *Multi-modal linguistic inference*, ***Logic Journal of Interest Group on Propositional Logic***, vol. 17 (1995), no. 3, pp. 555–585.

[5] A. S. Troelstra, H. Schwichtenberg, ***Basic Proof Theory***, Cambridge University Press, 2001.

▸ DENIS I. SAVELIEV, *A simple result on the continuum function.*
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We show for a wide class of ordinal functions F, if there exist parameters $\beta_1, \dots, \beta_n$ such that $2^{\aleph_\alpha} = \aleph_{F(\alpha,\beta_1,\dots,\beta_n)}$ for all α, then all the $\beta_1, \dots, \beta_n$ are finite.

▸ PHILIPP SCHLICHT, *Projective absoluteness and thin equivalence relations.*

Σ^1_n absoluteness for a forcing means that the same Σ^1_n facts about reals in the ground model are true in the ground model and in the generic extension. A related property is that there are no new equivalence classes of thin $\undertilde{\Pi}^1_{n-1}$ and $\undertilde{\Sigma}^1_{n-1}$ equivalence relations (equivalence relations with no perfect set of pairwise inequivalent reals) in the generic extension. We use an idea from Foreman and Magidor 1995 to show both properties for reasonable forcing if the right iterable models with Woodin cardinals exist. This can be applied to $\undertilde{\Sigma}^1_2$ c.c.c. forcing assuming projective determinacy.

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The given work continues works [1]–[8], devoted automatic recognition various properties in some classes of nonclassical logics.

In the given work the algorithms allowing to carry out automatic recognition of weak interpolation property (WIP) at logics extending K4 and programs realizing these algorithms are described.

Denote by $L + A$ the extension of logic L by an extra axiom scheme A.

Let us denote for $n \leq 1$

$\mathbf{S_n} = \{0, 1, \dots n\}$ where where xRy for all x, y;

The following proposition will help us to automate recognition of the WIP:

Proposition

Let $L = K4 + \{A_1, \dots, A_k\}$, $A = A_1 \wedge \dots \wedge A_k$, A contain n modalities and m variables. Then L has WIP if and only if one of the following conditions holds:

1. A is refuted in S_3;
2. A is true on $S_{\min(n+1,2^m)}$.

This proposition is turns out from Theorem 9 from [9] and Theorem 1 from [8].

The author creates the computer program realizing above described algorithm of recognition of the Weak Interpolation Property at modal calculus which contains logic $K4$.

This work has been supported by grant of Russian Foundation for Basic Research 06-01-00358.

[1] Schreiner P. *Automatic recognition of the interpolation property for some superintuitionistic propositional logics*, ***Vestnik of Novosibirsk State University***, vol. 3, no. 4 (2003), p. 85–92.

[2] Maksimova L. and Schreiner P., *Recognition algorithms of being tabular and pretabular in extensions of intuitionistic calculus*, ***Vestnik of Novosibirsk State University***, vol. 6, no. 3, (2006), p. 49–58.

[3] Schreiner P. *Automatized recognition of interpolation over S4.3*, ***Book of abstracts, Logic Colloquium 2005, Athens, Greece***, (2005), p. 113.

[4] Schreiner P. *Automatized recognition of the interpolation property in the extensions of the S5*, ***Abstracts, the 9th Asian Logic Conference***, (2005), p. 123–124.

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[7] Schreiner P. *Recognition of the Craig Interpolation Property and Projective Beth Property in the local tabular positive calculus*, ***Logic Colloquium 2006, Nijmegen, the Netherlands***, (2006), p. 35.

[8] Schreiner P. *Automatic recognition of interpolation in modal calculi*, ***Algebra and Logic***, vol. 46, no. 1 (2007), p. 62–70.

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Martin-Löf type theory (MLTT) is is based on the dependent function type and (inductively defined) algebraic types. In order to model concepts like interaction or object-orientation in MLTT in a direct way, it is useful to add (coinductively defined) weakly final coalgebras to MLTT. We introduce formation, introduction, elimination, and equality rules for weakly final coalgebras in MLTT. We will show that guarded induction is nothing but an informal description of the introduction rules for weakly final coalgebras.

We investigate the duality between algebraic and coalgebraic types in those rules: For algebraic types the introduction rules are simple and predicative, the elimination rules involve some degree of impredicativity. There is a large variety of possible elimination rules, all of which are derived from the principle of having a least set closed under the introduction rules. For coalgebraic types, the elimination rules are simple and predicative, whereas the introduction rules involve some degree of impredicativity. There is a large variety of possible introduction rules, all of which are derived from the principle of having the largest set allowing the elimination principle.

We introduce a model of the extension of MLTT by weakly final coalgebras, and investigate the implications for meaning explanations, namely the need for types, the meaning of which is given by an elimination principle.

We will then show that bisimulation is an example of a dependent weakly final coalgebra. We demonstrate that proofs by guarded induction of bisimulation is a much more intuitive way of proving bisimulation properties than the usual proofs based on the introduction of a bisimulation relation.

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Motivated by the classical Ramsey for pairs problem in reverse mathematics we investigate the recursion-theoretic complexity of certain assertions which are related to the Erdös-Szekeres theorem. We show that resulting density principles give rise to Ackermannian growth. We then parameterize these assertions with respect to a number-theoretic function f and investigate for which functions f Ackermannian growth is still preserved. Let d be a natural number and A_d be the dth approximation of the Ackermann function A. We show that Ackermannian growth is preserved for $f(i) = i^{\frac{1}{A^{-1}(i)}}$, but not for $f(i) = i^{\frac{1}{A_d^{-1}(i)}}$.

[1] Stephen G. Simpson, ***Subsystems of second order arithmetic***, Perspectives in Mathematical Logic, Springer-Verlag, Berlin, 1999, xiv+445.

[2] Peter A. Cholak, Carl G. Jockusch, and Theodore A. Slaman, *On the strength of Ramsey's theorem for pairs*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 1, pp. 1–55.

[3] Paul Erdös and George Szekeres, *A combinatorial problem in geometry*, ***Compositio Mathematica***, vol. 2 (1935), pp. 463–470.

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The notion of *Turing degree spectrum* of a countable structure $\mathfrak{A}$ is introduced by Richter as the set of all Turing degrees of the presentations of $\mathfrak{A}$. Soskov [1] initiated the study of the properties of the degree spectrum as a set of enumeration degrees. He considered *all* enumerations of the structure, not only the injective ones. The benefit of this definition is that every degree spectrum is upwards closed with respect to total enumeration degrees. The *co-spectrum* of $\mathfrak{A}$ is the set of all lower bounds of the elements of the degree spectrum of $\mathfrak{A}$. The degree spectra behave with respect to their co-spectra very much like the cones of the enumeration degrees $\{\mathbf{x} \mid \mathbf{x} \geq \mathbf{a}\}$ behave with respect to the ideals $\{\mathbf{x} \mid \mathbf{x} \leq \mathbf{a}\}$. Further properties true of the degree spectra but not necessarily true of all upwards closed sets are: the minimal pair theorem for the degree spectrum and the existence of quasi-minimal degree for the degree spectrum. These properties remain true for the relativized spectrum of a structure with respect to finitely many structures, introduced in [4].

We relativize Soskov's approach to degree spectra by considering multi-component spectra, i.e. a degree spectrum with respect to a given sequence of sets of natural numbers. We study this under the ω-enumeration reducibility. It is a uniform reducibility between sequences of sets of natural numbers, introduced and studied in [2, 3]. The notion of ω-degree spectrum generalizes the notion of relative spectrum. The ω-co-spectrum is the set of ω-enumeration degrees which are lower bounds of the elements of the ω-spectrum.

We prove that some properties of the degree spectrum such as the minimal pair theorem and the existence of quasi-minimal degree are true for the ω-degree spectrum. We give an explicit form of the elements of the ω-co-spectrum of a structure by means of recursive Σ_k^+ formulae.

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[2] Soskov, I. N., Kovachev, B., *Uniform regular enumerations*, ***Mathematical Structures in Computer Science*** vol. 16 (2006), no. 5, pp. 901–924

[3] Soskov, I. N., *The ω-enumeration degrees*, ***Journal of Logic and Computation***, vol. 17 (2007), no. 6, pp. 1193–1214

[4] Soskova, A. A., *Relativized degree spectra*, ***Journal of Logic and Computation*** vol. 17 (2007), no. 6, pp. 1215–1233

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Public announcement logics have been intensely studied in the last twenty years. The underlying language is normally given by the language of multi-agent modal logic augmented with dynamic-style operators for announcements. So public announcement logic is often called *dynamic epistemic logic*. That is for all formulas α, β we have that $[\alpha!]\beta$ is also a formula, which has the meaning *β holds after the public announcement of α*. We recommend to read the book of van Ditmarsch, van der Hoek, and Kooi [1] for a detailed introduction.

There are several approaches in literature in order to formalize public or private communication in the language of dynamic epistemic logic. The well-known *truthful public announcements* can be added to typical systems of knowledge like $\mathsf{S5}_n$, cf. [1]. Other definitions allow private communication called *group announcements* and are compatible with various systems of belief. We confine ourselves to mentioning the approach of Gerbrandy and Groeneveld [2] for $\mathsf{K45}_n$, and our consistency preserving approach [4] for $\mathsf{KD45}_n$.

Although there are many different contributions to public announcements in literature, there is no approach combining knowledge, belief, and public announcements. One of the reasons for this lacking is that the standard public announcements are partial, see Proposition 4.11 of [1]. That is, the announcement with a false formula results in an inconsistent state. Using our *total public announcements* [5], we are able to present bimodal systems augmented with public announcement operators. In our setting, truthful announcements affect both knowledge and belief of an agent. On the other hand, a false announcement doesn't directly affect an agent's knowledge, because false information cannot be known.

There are many possible ways to combine knowledge and belief, see [3] for an overview. We choose two maximal systems not satisfying undesired properties like

$$B_i\alpha \leftrightarrow K_i\alpha, \qquad\qquad B_iK_i\alpha \to \alpha,$$

and we define an announcement semantics for both of these systems. Moreover, we give axiomatizations and prove soundness and completeness.

[1] Hans van Ditmarsch, Wiebe van der Hoek, and Barteld Kooi, ***Dynamic Epistemic Logic***, vol. 337 of Synthese Library, Springer, 2007.

[2] Jelle Gerbrandy and Willem Groeneveld, *Reasoning about information change*, ***Journal of Logic, Language and Information***, vol. 6 (1997), no. 2, pp. 147–169.

[3] Wiebe van der Hoek, *Systems for knowledge and belief*, ***Journal of Logic and Computation***, vol. 3 (1993), no. 2, pp. 173–195.

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► R. GREGORY TAYLOR, *Symmetric propositions and second-order logic.*
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Given extended Zermelo system $\mathcal{H}_{\mathfrak{D},\Sigma}$ over domain $\mathfrak{D}$, we describe second-order language $\mathcal{L}^{\mathrm{SOL}}_{\mathfrak{D},\Sigma}$ with constants for arbitrary relations over $\mathfrak{D}$ and define the *canonical expansion modulo saturation* of any formula of $\mathcal{L}^{\mathrm{SOL}}_{\mathfrak{D},\Sigma}$ having no occurrences of individual or function constants. We show that the canonical expansion of any sentence of $\mathcal{L}^{\mathrm{SOL}}_{\mathfrak{D},\Sigma}$ having no occurrences of individual or function constants is symmetric in Zermelo's sense. A converse is presented as well. The relation between second-order logic and Zermelo's theory of systems of infinitely long propositions is thereby clarified.

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The region-based theory of space goes back to Whitehead but last decade it draws additional attention from the so called qualitative spatial reasoning. It is the first-order theory of the class of contact Boolean algebras, i.e. Boolean algebras of regular closed sets in connected normal topological spaces with the binary relation C, contact, defined with $C(a,b) \iff a \cap b \neq \emptyset$. In [1] the universal fragment T of this theory is investigated as fragment L of an appropriated modal logic. Here we prove that T coincides with the universal fragment of some narrow class of structures: contact Boolean algebras of the class of topological spaces of the type (X,τ), where X is finite union of squares in $\mathbb{R}^2$ and τ is the induced topology. We demonstrate also non-canonicity of L.

[1] Balbiani, Ph., T. Tinchev, D. Vakarelov, *Modal logics for region-based theories of space. Fundamenta Informaticae, 81(1-3): 29-82, 2007.*, ***Fundamenta Informaticæ***, vol. 81 (2007), no. 1–3, pp. 29–82.

► TRIFON TRIFONOV, *Simulating modified realizability and A-translation with Gödel's Dialectica interpretation.*

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Recently Hernest and Oliva [5] showed how Gödel's functional (Dialectica) interpretation [3] can be combined with Kreisel's modified realisability [7] when considered in a linear logic setting. An alternative approach was taken up by Hernest and the author [6], based on Berger's uniform quantifiers [1] with Hernest's extension to the Dialectica interpretation [4]. We showed that in an intuitionistic logic setting positive and negative computational contributions of quantified variables can be isolated in a sound way. Here we discuss how this approach can be further extended to implication, allowing to simulate modified realisability by fully suppressing negative content in the functional interpretation. As a result we are also able to model a refined variant [2] of Dragalin/Friedman's A-translation.

[1] Ulrich Berger, *Uniform Heyting arithmetic*, ***Annals of Pure and Applied Logic***, vol. 133 (2005), no. 1–3, pp. 125–148.

[2] Ulrich Berger, Wilfried Buchholz and Helmut Schwichtenberg, *Refined program extraction form classical proofs*, ***Annals of Pure and Applied Logic***, vol. 114 (2002), no. 1–3, pp. 3–25.

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[4] Mircea-Dan Hernest, *Light functional interpretation*, ***19th International Workshop, CSL 2005*** (Oxford, UK), vol. 3634, Lecture Notes in Computer Science, Springer, 2005, pp. 477–4492.

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► ALEXEY G. VLADIMIROV, *Kripke models for intuitionistic set theory.*

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Let $ZFI2C$ be usual first order intuitionistic Zermelo-Fraenkel set theory with scheme $Collection$ in two-sorted language. Axioms and rules of this system are: all usual axioms and rules of intuitionistic predicate logic HPC, all usual axioms of intuitionistic arithmetic (HA) for variables of sort 0, and all usual axioms and schemes of Zermelo-Fraenkel system for variables of sort 1. Kripke models are used intensively for investigations of intuitionistic arithmetic and analysis ([1]),[2]). For set theory Kripke models were used, for example, in [3] to prove that $ZFI2C$ does not have the full existence property, and that classes of provably recursive functions in $ZFI2C$ and in $ZFI2R$ (with Replacement without Collection) are different, so, $ZFI2C$ is stronger than $ZFI2R$. It is well-known that Smorinsky operation over Kripke models for Heyting arithmetic HA permit to get a lot of results about HA (see [1], [2]). But for Kripke models for set theory it is not clear, how to define an analog of this operation, because in the case of HA it is very important that for each HA- model Kripke there a is common part of all domains, but in case of $ZFI2C$-models Kripke it is not clear, what would be a such common part (see [1]) for discussion of Smorinsky operation for HA-models).

We consider Kripke models for this theory in some generalized sense in the style of [1]. This let us work without using Smorinsky operation for Kripke models for set theory.

Using this semantic we prove the following statement for $ZFI2C$ and for $ZFI2C + DCS$ the following statements:

1. DP without parameters implies EP without parameters (some variant of well-known result of H.Friedman; DP for $ZFI2C$ was proved in [4]):
2. Admissibility of Markov‘s rule without parameters;
3. Underivability of Weak Markov principle;
4. Underivability of the principle P;
5. Admissibility of the rule P without parameters;
6. Underivability of the Least Number Principle.

All these results can be generalized to every set theory $T = ZFI2C + \Gamma$ such that if K is a Kripke model for T then K' is a Kripke model for T. For example, this is true for $ZFI2C + DCS$.

[1] A.G. Dragalin. *Mathematical intuitionism. Introduction to the Proof Theory*, 1988. (Translations of Mathematical Monographs, vol. 67)

[2] A.S.Troelstra Metamathematical investigations of intuitionistic arithmetic and analysis. Lecture Notes in mathematics, 1973, vol. 344.

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[4] A.G.Vladimirov Effectivity properties for intuitionistic set theory with scheme Collection. In appear.

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Locally countable upper semi-lattices (usl) of cardinality ω_1 were characterized by Abraham and Shore [1] as initial segments of Turing degrees. However Groszek and Slaman [2] built a model of ZFC which contains a locally countable usl of cardinality 2^ω not embeddable into Turing degrees, and conjectured that the answer is always positive under Martin's Axiom (MA).

In this paper we confirm Groszek-Slaman's conjecture by embedding every locally countable usl of cardinality 2^ω into Turing degrees. To this end we introduce embeddings which map incomparable elements in usl to sets relatively hyperimmune in each others and apply MA to inductively extend such embeddings.

[1] Uri Abraham, Richard A. Shore, *Initial segments of the degrees of size* $\aleph_1$, ***Israel Journal of Mathematics***, vol. 53(1986), no. 1, pp.1–51.

[2] Marcia J. Groszek, Theodore A. Slaman, *Independence results on the global structure of the Turing degrees*, ***Transactions of American Mathematical Society***, vol. 277(1983), no. 2, pp.579–587.

► RICARDO WEHBE, *A cut-free axiomatization for relativized common knowledge.*
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We present an axiomatization for *Relativized Common Knowledge* [4] which is a generalization of Common Knowledge [2]. Our work follows previous work on Temporal Logics of Martin Lange and Kai Brünnler [1] and Colin Stirling [3]. The axiomatization has several good properties, but also some drawbacks. We briefly describe them and point out some open problems.

[1] Brünnler, Kai; Lange, Martin *Cut-Free Systems for Temporal Logic*, to appear in ***The Journal of Logic and Algebraic Programming***.

[2] Fagin, Ronald; Halpern, Joseph; Moses, Yoram; Vardi, Moshe ***Reasoning About Knowledge***, The MIT Press, Cambridge, MA, 1996.

[3] Lange, Martin; Stirling, Colin *Focus Games for Satisfiability and Completeness of Temporal Logic*, ***Proc. of the 16th Symp. on Logic in Computer Science (LICS'01)***, 2001, pp. 357–365.

[4] van Benthem, Johan; van Eijck, Jan; Kooi, Barteld ***Logics of Communication and Change, Information and Computation 204***, 2006, pp. 1620-1662.

▸ STEFAN WINTEIN, *A riddle about truth*.

In this paper, I analyze a semantic riddle: there are two brothers, one lying, one speaking the truth and both brothers know that p. You do not know p, nor which brother is the liar. Can you come up with a single question that allows you to determine whether p is the case? I model the intuitive reasoning we use in solving the riddle by a derivation in first order logic from some axiom set K. As K contains axiom schemes closely resembling the well-known T-scheme and as we formulate K in a semantically closed language, inconsistency is on the lure. I show that K is consistent by constructing a model using a *revision process*. However, extending K with the axioms of Robinson's Arithmetic does result in an inconsistent system. Then I point out an equivalence between the semantic riddle and a quantum computational problem. First, I show that "solving the riddle" comes down to determining the parity of an unknown function f on the set $\{0, 1\}$ in just one query. This problem cannot be solved using classical, but can be solved using quantum computation: it is known as *Deutsch's problem*.

▸ MARS M. YAMALEEV, *Low c.e. and low 2-c.e. degrees are not elementarily equivalent*.

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Given Turing degrees $\mathbf{0} < \mathbf{b} < \mathbf{a}$ and a class of Turing degrees $\mathcal{C}$, we say that $\mathbf{b}$ *is noncuppable to* $\mathbf{a}$ *in the class* $\mathcal{C}$ if there is no degree $\mathbf{w} \in \mathcal{C}$ such that $\mathbf{w} < \mathbf{a}$ and $\mathbf{a} = \mathbf{b} \cup \mathbf{w}$; we say that $\mathbf{b}$ *is strongly noncuppable to* $\mathbf{a}$ *in the class* $\mathcal{C}$ if there is no degree $\mathbf{w} \in \mathcal{C}$ such that $\mathbf{a} \not\leq \mathbf{w}$ and $\mathbf{a} \leq \mathbf{b} \cup \mathbf{w}$.

Let $\mathbf{R}^{\mathbf{low}}$ and $\mathbf{D}_{\mathbf{2}}^{\mathbf{low}}$ be the classes of all low computably enumerable (c.e.) and all low 2-c.e. degrees, respectively. We proved the following theorem.

Theorem 1. *There exist noncomputable low c.e. degrees* $\mathbf{b} < \mathbf{a}$ *such that* $\mathbf{b}$ *is strongly noncuppable to* $\mathbf{a}$ *in the class* $\mathbf{R}^{\mathbf{low}}$ *and for any low degree* $\mathbf{w}$ *the degree of* $\mathbf{b} \cup \mathbf{w}$ *is low again.*

Together with the following result this gives that the partial orders $\mathbf{R}^{\mathbf{low}}$ and $\mathbf{D}_{\mathbf{2}}^{\mathbf{low}}$ are not elementarily equivalent.

Theorem (Cooper, Lempp, Watson[1]). *For every high c.e. degree* $\mathbf{h}$ *and for every noncomputable n-c.e.* ($n \geq 1$) *degree* $\mathbf{b} < \mathbf{h}$ *there exists a low 2-c.e. degree* $\mathbf{d}$ *such that* $\mathbf{h} = \mathbf{b} \cup \mathbf{d}$.

[1] Cooper S.B., Lempp S., and Watson P., *Week density and cupping in the d-c.e degrees.*, ***Israel Journal of Mathematics***, vol. 67 (1989), pp. 137–152.

▸ XUNWEI ZHOU, *Rule-based inference is suitable for human while hypothetical inference is suitable for computer*.

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Take "if triangle ABC is congruent with triangle $A'B'C'$, then side AB equals side $A'B'$ " as an example. Rule-based inference is described by

$$\frac{\Delta ABC \equiv \Delta A'B'C'}{AB = A'B'}$$

where the general law $\Delta ABC \equiv \Delta A'B'C' \rightarrow AB = A'B'$ is used as an inference rule, inferring the unknown fact $AB = A'B'$ from the known fact $\Delta ABC \equiv \Delta A'B'C'$. Hypothetical inference is described by

$$\frac{\begin{array}{c}\Delta ABC \equiv \Delta A'B'C' \rightarrow AB = A'B' \\ \Delta ABC \equiv \Delta A'B'C'\end{array}}{AB = A'B'}$$

where the general law $\Delta ABC \equiv \Delta A'B'C' \rightarrow AB = A'B$ is used as a geometrical theorem, taken as the major premise, $\Delta ABC \equiv \Delta A'B'C'$ is taken as the minor premise, inferring the conclusion $AB = A'B'$. In rule-based geometry system, there are many inference rules, a computer cannot choose from them automatically, but a human can. Therefore, rule-based inference is not suitable for computer but is suitable for human. Geometrical hypothetical inference system transforms all these inference rules into geometrical theorems, taking hypothetical inference as the sole inference rule. Because there is only one inference rule, every time a computer wants to make inference, the computer uses it. As to the many geometrical theorems and facts, the computer searches them top-down, from left to right, depth-first plus backtracking, like the way Prolog does. In this way, the inference can be made automatic. So, hypothetical inference is suitable for computer. But because there are too many repetitive work, it is not suitable for human.

[1] Xunwei Zhou, Rule-based inference v. hypothetical inference, 2008 ASL Annual Meeting, Irvine, California, USA, March 27-30, 2008▸ ALBERT ZIEGLER, *Heyting models and realizability models for constructive set theory*.

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Heyting models are a generalisation of the well examined Boolean models (a forcing method) for classical set theory whereas realizability is a typically constructive method of obtaining models with very unclassical features. There is however a way to obtain a common generalisation, forcing with a formal topology blended with an applicative structure. This not only increases our understanding of class models of set theory, but actually it makes several new results available which have until now only been known to hold about one of the two types of models or which have not been examined. For example, this method provides relatively simpe proofs that the relation reflection scheme or regular extension axioms are absolute for both types of models. Also it enables models which are of none of the known types and show new independence results about constructive set theory that could not have been obtained with the previously normally used machinery.

▸ MAXIM ZUBKOV, *On eta-representable sets*.

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I will talk about a classification of Turing degrees with strongly η-representable sets.

In particularly, I'll show that if A is $\Sigma_{\omega}^{-1,\,\emptyset}$ set then A is strongly η-representable. On the over hand $A \in \Sigma_{\omega}^{-1,\,\emptyset}$ provided that a set $A \oplus \omega$ is strongly η-representable.

An infinite set $A = \{a_0 < a_1 < \dots\}$ is called strongly η-representable, if there is a computable linear ordering of order type $\eta + a_0 + \eta + a_1 + \dots$.

By definition a set A is $\Sigma_{\omega}^{-1,\,\emptyset}$ if and only if there is a computable sequence of computable functions $f_i(x,\, s)$, such that

1) $\chi_{A_i}(x) = lim\, inf_s f_i(x,\, s)$;

2) $A_0 \subseteq A_1 \subseteq A_2 \subseteq \dots$

3) $A = \bigcup\limits_{i=0}^{\infty} (A_{2i+1} - A_{2i})$.